SEIBERG-WITTEN NONCOMMUTATIVE MINIMAL SUPERSYMMETRIC STANDARD MODEL

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The noncommutative supersymmetric transformations for minimal standard model (MSSM) is derived by using Seiberg-Witten map. The action of noncommutative MSSM for component fields is carried out.

Key words: supersymmetric field theories, noncommutative geometry, models beyond the standard models.

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1. INTRODUCTION

During the last decade, there was a growing interest in noncommutative field theories. The noncommutativity of space-time was shown explicitly in physical systems such as Landau levels and D-branes in the presence of B-field [1].

In the noncommutative space-time theories, the commutator of the space-time coordinates is proportional to some antisymmetric constant parameter. Those theories turned up to be non local and therefore nonrenormalizable. In an attempt to remedy to this problem, Doplicher *et al.* [2] proposed a new algebra where the noncommutative parameter is promoted to be an antisymmetric operator. Armoni showed that the $U(N)$ gauge theories make possible to calculate the gluonic propagators to the first order, which was not possible in the $SU(N)$ case [3]. These problems have been overcome by Jurco *et al.*, it who used Moyal star product to construct non abelian gauge theories, namely the noncommutative standard model [4]. Recently Dayi *et al.* [5] built a noncommutative supersymmetric $U(1)$ model by generalizing Seiberg-Witten map [6].

Following [5], we propose, using this approach, a *Noncommutative Supersymmetric Standard Model* (NCMSSM). In section two, we show the main features of *Minimal Supersymmetric Standard Model* (MSSM). We present a brief review of the Seiberg-Witten map in the third section. In section four, we give the details of the various fields transformations in the contest of the Seiberg-Witten noncommutative space-time geometry and the resulted action with various new interactions.

2. MINIMAL SUPERSYMMETRIC STANDARD MODEL

The *Minimal Supersymmetric Standard Model* (MSSM) was built starting from the gauge invariance under the two following groups transformations: $U(1)$ for the hypercharge and $SU(2)$ for the left symmetry. It's the theoretically best motivated and conceptually most elaborated and predictive framework beyond the *Standard Model* (SM). All the fundamental particles of spin 1/2 or 0 have supersymmetric partners of spin 0 and 1/2 respectively. One will assign with the particles small letters whereas their super partners capital letters. An overview of the component fields is given in Table 2.1 [7].

Using the Weyl notations and superspace formalism, the supersymmetric kinetic and interaction terms of the classical action denoted by $S_{Kinetic}$ and $S_{Interaction}$ is written:

$$S_{MSSM} = S_{Kinetic} + S_{Interaction}, \tag{1}$$

With:

$$\begin{aligned} S_{kinetic} = & \frac{1}{16}\sum_i \int d^8 z \overline{H_i} e^{2gV+g'YV'} H_i + \frac{1}{16}\int d^8 z \overline{L} e^{2gV+g'YV'} L + \frac{1}{16}\int d^8 z \overline{R} e^{g'YV'} R + \\ & + \frac{1}{16}\int d^8 z \overline{Q} e^{2gV+g'YV'} Q + \frac{1}{16}\int d^8 z \overline{U} e^{g'YV'} U + \frac{1}{16}\int d^8 z \overline{D} e^{g'YV'} D + \\ & - \frac{1}{512}\int d^6 z 2tr\left[F^\alpha F_\alpha\right] - \frac{1}{128}\int d^6 z 2\left[F'^\alpha F'_\alpha\right]. \end{aligned} \tag{2}$$

Table 2.1

Multiplets and field content of the theory

	Bosonic fields	Fermionic fields	Isospen	Hypercharge
Gauge multipletes	V^a V'	λ^a λ'	triplet singlet	0 0
Higges fields	$H_1 = (H_1^1, H_1^2)^t$ $H_2 = (H_2^1, H_2^2)^t$	$h_1 = (h_1^1, h_1^2)^t$ $h_2 = (h_2^1, h_2^2)^t$	doublet doublet	–1 1
Leptons	$L = (L^1, L^2)^t$ R	$l = (l^1, l^2)^t$ r	doublet singlet	–1 2
Quarks	$Q = (Q^1, Q^2)^t$ U D	$q = (q^1, q^2)^t$ u d	doublet singlet singlet	1/3 –4/3 2/3

For the measure notations $\int d^8z$, $\int d^6z$ and the superfields strengths F^α and F'^α expressions, see appendix 6. Here g, g' and Y represent the coupling constants and the weak hypercharge respectively. The trace *(tr)* is over the group index.

$$S_{interaction} = -\frac{f_R}{4}\int d^6zH_1^t\left(i\sigma_2\right)LR - \frac{f_U}{4}\int d^6zH_2^t\left(i\sigma_2\right)QU - \\ -\frac{f_D}{4}\int d^6zH_1^t\left(i\sigma_2\right)QD + \frac{\mu}{4}\int d^6zH_1^t\left(i\sigma_2\right)H_2 + c.c, \tag{3}$$

Here $c.c$ stands for the complex conjugate and f_R, f_D, μ interactions constants.

The superfield expressions in terms of component fields are given by:

$$V = \theta^\alpha\sigma_{\alpha\dot{\alpha}}{}^\mu\bar{\theta}^{\alpha\dot{\alpha}}V_\mu + \overline{\theta^2}\theta^\alpha\lambda_\alpha + \theta^2\overline{\theta_{\dot{\alpha}}}\overline{\lambda^{\dot{\alpha}}} + \frac{1}{2}\theta^2\bar{\theta}^2D_V, \tag{4}$$

$$V' = \theta^\alpha\sigma_{\alpha\dot{\alpha}}{}^\mu\bar{\theta}^{\alpha\dot{\alpha}}V'_\mu + \overline{\theta^2}\theta^\alpha\lambda'_\alpha + \theta^2\overline{\theta_\alpha}\lambda'^{\dot{\alpha}} + \frac{1}{2}\theta^2\bar{\theta}^2D_{V'}, \tag{5}$$

$$L = e^{-i\theta\sigma^\mu\overline{\theta\partial_\mu}}\left(L + \sqrt{2}\theta^\alpha l_\alpha + \theta^2F_L\right), \tag{6}$$

$$R = e^{-i\theta\sigma^\mu\overline{\theta\partial_\mu}}\left(R + \sqrt{2}\theta^\alpha r_\alpha + \theta^2F_R\right), \tag{7}$$

$$Q = e^{-i\theta\sigma^\mu\overline{\theta\partial_\mu}}\left(Q + \sqrt{2}\theta^\alpha q_\alpha + \theta^2F_Q\right), \tag{8}$$

$$U = e^{-i\theta\sigma^\mu\overline{\theta\partial_\mu}}\left(U + \sqrt{2}\theta^\alpha u_\alpha + \theta^2F_U\right), \tag{9}$$

$$D = e^{-i\theta\sigma^\mu\overline{\theta\partial_\mu}}\left(D + \sqrt{2}\theta^\alpha d_\alpha + \theta^2F_D\right), \tag{10}$$

$$H_i = e^{-i\theta\sigma^\mu\overline{\theta\partial_\mu}}\left(H_i + \sqrt{2}\theta^\alpha h_{i\alpha} + \theta^2F_i\right). \tag{11}$$

θ_α is a Grassman variable. More notations for the super-space and gauge group generators can be found in appendix 6.

Now we can write the whole expressions of kinetic and interactions parts (2) and (3) of the classical action in terms of component fields using equations (4)–(11) and after integration over the superspace coordinates θ_α:

$$S_{kinetic} = \int d^4x\Big[(\overline{D_\mu L}D^\mu + il^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\overline{D_\mu l}^\alpha + \overline{F_L}F_L + g\bar{L}D_VL + g'\frac{Y}{2}\bar{L}D_{V'}L - \\ -(\sqrt{2}g\bar{l}_{\alpha\dot{\alpha}}\bar{\lambda}^{\alpha\dot{\alpha}}L + \sqrt{2}g'\frac{Y}{2}\bar{l}_{\alpha\dot{\alpha}}\bar{\lambda}'^{\alpha\dot{\alpha}}L + c.c)\Big] +$$

$$
\begin{aligned}
&+\int d^4x\Big[\overline{D_\mu R}D^\mu R+ir^\alpha\sigma^\mu_{\alpha\dot\alpha}\overline{D_\mu r}^{\alpha\dot\alpha}+\overline{F_R}F_R+g'\frac{Y}{2}\overline{R}D_V R-\\
&-(\sqrt{2}g'\frac{Y}{2}\overline{r}_{\alpha\dot\alpha}\overline{\lambda'}^{\alpha\dot\alpha}R+c.c)\Big]+\\
&+\int d^4x\Big[\overline{D_\mu Q}D^\mu Q+iq^\alpha\sigma^\mu_{\alpha\alpha}\overline{D_\mu q}^{\alpha\dot\alpha}+\overline{F_Q}F_Q+g\overline{Q}D_V Q+g'\frac{Y}{2}\overline{Q}D_{V'}Q-\\
&\qquad-(\sqrt{2}g\overline{q}_{\tilde\alpha\dot\alpha}\overline{\lambda}^{\alpha\dot\alpha}Q+\sqrt{2}g'\frac{Y}{2}\overline{q}_{\dot\alpha}\overline{\lambda'}^{\alpha\dot\alpha}Q+c.c)\Big]+\\
&+\int d^4x\Big[\overline{D_\mu U}D^\mu U+iu^\alpha\sigma^\mu_{\alpha\dot\alpha}\overline{D_\mu}u^{\dot\alpha}+\overline{F_U}F_U+g'\frac{Y}{2}\overline{U}D_{V'}U-\\
&-(\sqrt{2}g'\frac{Y}{2}\overline{u}_{\tilde\alpha\dot\alpha}\overline{\lambda'}^{\alpha\dot\alpha}U+c.c)\Big]+\\
&+\int d^4x\Big[\overline{D_\mu D}D^\mu D+id^\alpha\sigma^\mu_{\alpha\dot\alpha}\overline{D_\mu d}^{\alpha\dot\alpha}+\overline{F_D}F_D+g'\frac{Y}{2}\overline{D}D_{V'}D-\\
&-(\sqrt{2}g'\frac{Y}{2}\overline{d}_{\tilde\alpha\dot\alpha}\overline{\lambda'}^{\alpha\dot\alpha}D+c.c)\Big]+\\
&+\sum_i\int d^4x\Big[(\overline{D_\mu H_i}D^\mu H_i+ih_i^\alpha\sigma^\mu_{\alpha\dot\alpha}\overline{D_\mu h_i}^{\alpha\dot\alpha}+\overline{F_i}F_i+g\overline{H}D_V H_i+\\
&+g'\frac{Y}{2}\overline{H}_i D_{V'}H_i-(\sqrt{2}g\overline{h_i}_{\alpha\dot\alpha}\overline{\lambda}^{\alpha\dot\alpha}H_i+\sqrt{2}g'\frac{Y}{2}\overline{h_i}_{\alpha\dot\alpha}\overline{\lambda'}^{\alpha\dot\alpha}H_i+c.c)\Big]+\\
&+\int d^4x\,2tr\Big(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+i\lambda^\alpha\sigma^\mu_{\alpha\dot\alpha}\overline{D_\mu\lambda}^{\alpha\dot\alpha}+\frac{1}{2}D_V D_V\Big)+\\
&+\int d^4x\,2tr\Big(-\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu}+i\lambda'^\alpha\sigma^\mu_{\alpha\dot\alpha}\overline{D_\mu\lambda'}^{\alpha\dot\alpha}+\frac{1}{2}D_{V'}D_{V'}\Big),
\end{aligned}
\tag{12}
$$

The interaction terms is:

$$
\begin{aligned}
S_{interaction}=f_R&\int d^4x[-H_1^t(i\sigma_2)lr+H_1^t(i\sigma_2)F_R R+H_1^t(i\sigma_2)LF_R-\\
&-h_1^{t\alpha}(i\sigma_2)l_\alpha R-h_1^{t\alpha}(i\sigma_2)Lr_\alpha+F_1^t(i\sigma_2)LR]+\\
&+f_U\int d^4x[-H_2^t(i\sigma_2)qu+H_2^t(i\sigma_2)F_U U+H_2^t(i\sigma_2)QF_U-\\
&-h_2^{t\alpha}(i\sigma_2)q_\alpha U-h_2^{t\alpha}(i\sigma_2)Qu_\alpha+F_2^t(i\sigma_2)QU]+\\
&+f_D\int d^4x[-H_1^t(i\sigma_2)qd+H_1^t(i\sigma_2)F_D D+H_1^t(i\sigma_2)QF_D-\\
&-h_1^{t\alpha}(i\sigma_2)q_\alpha D-h_1^{t\alpha}(i\sigma_2)Qd_\alpha+F_1^t(i\sigma_2)QD]+\\
&+\mu\int d^4x[-H_1^t(i\sigma_2)F_2+F_1^t(i\sigma_2)H_2+h_1^{t\alpha}(i\sigma_2)h_{2\alpha}]+c.c.
\end{aligned}
\tag{13}
$$

where $D_V,\ D_{V'},\ F_L, F_R, F_Q, F_U, F_D, F_i$ represent the auxiliary fields and the $\sigma^\mu_{\alpha\dot\alpha}$ is index structure of the Pauli matrices can be found in Appendix 6.

3. SEIBERG-WITTEN MAP

The Moyal-Weyl star product defined by a formal power series expansion of [4, 6, 9]:

$$(f*g)(x)=\exp\left(\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^{\mu}}\frac{\partial}{\partial y^{\nu}}\right)f(x)g(y)\Big|_{y\to x}. \tag{14}$$

By partial integration, one can show the property:

$$\int d^n x(f*g)(x)=\int d^n x(g*f)(x)=\int d^n x f(x)g(x), \tag{15}$$

$$\int d^n x f(x)*g(x)*h(x)=\int d^n x(f(x)*g(x))h(x)=\int d^n x f(x)(g(x)*h(x)). \tag{16}$$

This star product of ordinary functions f and g can be seen as a tower built upon its classical limit, which is determined by a Poisson tensor $\theta^{\mu\nu}$:

$$f*g=fg+\frac{i}{2}\theta^{\mu\nu}\partial_{\mu}f\partial_{\nu}g+O(\theta^2). \tag{17}$$

Generalization of the Seiberg-Witten map to noncommutative supersymmetric gauge theories can be formulated in some different ways. One of these is to generalize the definition of the map between the noncommutative gauge field $\hat{V}$, noncommutative matter field $\hat{\psi}$ and noncommutative gauge parameter $\hat{\Lambda}$ and the ordinary ones Λ, ψ, to $\hat{V}(V)$, $\hat{\psi}(\psi,V)$ and $\hat{\Lambda}(\Lambda,V)$ such that

$$\hat{V}(V)+\hat{\delta}_{\hat{\Lambda}}\hat{V}(V)=\hat{V}(V+\delta_{\Lambda}V), \tag{18}$$

$$\hat{\psi}(\psi,V)+\hat{\delta}_{\hat{\Lambda}}\hat{\psi}(\psi,V)=\hat{\psi}(\psi+\delta_{\Lambda}\psi). \tag{19}$$

Where δ_{Λ} is a ordinary gauge transformation and the $\hat{\delta}_{\hat{\Lambda}}$ is a noncommutative gauge transformation and are defined by:

$$\begin{cases}\hat{\delta}_{\hat{\Lambda}}\hat{V}_{\mu}=i\partial_{\mu}\hat{\Lambda}+i\left[\hat{\Lambda}^{*},\hat{V}_{\mu}\right], & \delta_{\Lambda}V_{\mu}=i\partial_{\mu}\Lambda,\\ \hat{\delta}_{\hat{\Lambda}}\hat{\psi}=i\hat{\Lambda}*\hat{\psi} & \delta_{\Lambda}V_{\mu}=i\Lambda\psi.\end{cases} \tag{20}$$

We first work to first order in θ. We write $\hat{V}=V+V'(V)$, $\hat{\psi}=\psi+\psi'(\psi,V)$ and $\hat{\Lambda}=\Lambda+\Lambda'(\Lambda,V)$, with V', ψ' and Λ' local function of Λ, ψ and V of order θ.

Equations (18–20) are solved by [6, 4, 10]:

$$\hat{V}_{\mu}=V_{\mu}+V'_{\mu}(V)=V\mu+\frac{1}{4}\theta^{\sigma\rho}\left\{V_{\rho},\partial_{\sigma}V_{\mu}+F_{\sigma\mu}\right\}+O(\theta^2), \tag{21}$$

$$\begin{aligned}\hat{\psi} &= \psi + \psi'(V,\psi) = \psi + \frac{1}{2}\theta^{\mu\nu}\rho_\psi(V_\nu)\partial_\mu\psi + \\ &+ \frac{i}{8}\theta^{\mu\nu}[\rho_\psi(V_\mu),\rho_\psi(V_\nu)]\psi + O(\theta^2),\end{aligned} \tag{22}$$

$$\hat{\Lambda}\rho = \Lambda + \Lambda'(\Lambda, V) = \Lambda + \frac{1}{4}\theta^{\sigma\rho}\left\{\partial_\sigma\Lambda, V_\rho\right\} + O(\theta^2). \tag{23}$$

At the first order in the noncommutative parameter $\theta^{\rho\sigma}$.

4. NONCOMMUTATIVE MINIMAL SUPERSYMMETRIC STANDARD MODEL

In Noncommutative Minimal Supersymmetric Standard Model (NCMSSM) construction, we use the same group as MSSM, $SU(2)_L \times U(1)_Y$.

The difference lies in the definitions of the various fields (to keep gauge invariance) and products. The most natural way is to take the classical tensor product and consider the whole gauge potential V_μ as defined by [5, 8]:

$$\hat{V}_\mu = g'Y\hat{V}'_\mu + g\hat{V}_{\mu a}T_L^a. \tag{24}$$

and the commutative gauge parameters Λ by:

$$\hat{\Lambda} = g'Y\hat{\alpha} + g\hat{\alpha}_a^L T_L^a. \tag{25}$$

Here Y and T_L^a are the generators of $U(1)_Y$ and $SU(2)_L$ respectively.

According to Seiberg-Witten map, the noncommutative gauge parameter $\hat{\Lambda}$ is given by:

$$\hat{\Lambda} = \Lambda + \Lambda^1, \tag{26}$$

Where the additional term has the following expression:

$$\Lambda^1 = \frac{1}{4}\theta^{\mu\nu}\left\{V_\nu, \partial_\mu\Lambda\right\} + O\left(\theta^2\right) \tag{27}$$

$\theta^{\mu\nu}$ is the noncommutative tensor.

Regarding the noncommutative fermion's fields $\hat{\psi}^{(n)}$ corresponding to particles labelled by (n), we can write:

$$\hat{\psi}^{(n)} = \psi^{(n)} + \psi^{(n)1}, \tag{28}$$

Here we write the additional noncommutative part as:

$$\psi^{(n)1} = \frac{1}{2}\theta^{\mu\nu}\rho_{(n)}(V_\nu)\partial_\mu\psi^{(n)} + \frac{i}{8}\theta^{\mu\nu}\left[\rho_{(n)}(V_\mu),\rho_{(n)}(V_\nu)\right]\psi^{(n)} \tag{29}$$

where $\rho_{(n)}(V\nu)$ is the matrix representation of field.

Similarly, the Seiberg-Witten map for the noncommutative vector potential $\hat{V}_\mu$ takes the form:

$$\hat{V}_\mu = V_\mu + V_\mu^1 \tag{30}$$

$$V_\mu^1 = \frac{1}{4}\theta^{\rho\sigma}\left\{V_\sigma, \partial_\rho V_\mu\right\} + \frac{1}{4}\theta^{\rho\sigma}\left\{F_{\rho\mu}, V_\sigma\right\} + O(\theta^2). \tag{31}$$

where $F_{\mu\nu}$ is the field strength.

And for the scalar field $\hat{\phi}$ we have:

$$\hat{\phi} = \phi + \phi^1, \tag{32}$$

$$\phi^1 = \frac{1}{2}\theta^{\mu\nu}\rho_{(n)}(V_\nu)\partial_\mu\phi + \frac{i}{4}\theta^{\mu\nu}\rho_{(n)}(V_\mu)\rho_{(n)}(V_\nu)\phi \tag{33}$$

Finally, the noncommutative auxiliary field D is redefined as:

$$\hat{D} = D - \theta^{\mu\nu}V_\mu\partial_\nu D \tag{34}$$

Using the new expressions of the various fields and by replacing the ordinary product with the Moyal * one (see section 3). The action can be expressed in a very compact way as:

$$S_{NCMSSM} = S_{Higgs} + S_{Matter,leptonic} + S_{Matter,quark} + S_{Gauge} + S_{interaction} \tag{35}$$

We define now each term in this expression starting with the non-commutative supersymmetric kinetic Higgs sector:

$$\begin{aligned} S_{Higgs} = \sum_i \int d^4x \Big[& \overline{\hat{D}_\mu \hat{H}_i} * \hat{D}^\mu \hat{H}_i + i\hat{h}_i^\alpha * \sigma^\mu_{\alpha\dot{\alpha}} \overline{\hat{D}_\mu \hat{h}_i}^{\alpha\dot{\alpha}} + \\ & + \overline{\hat{F}_i} * \hat{F}_i + g\overline{\hat{H}}_i * \hat{D}_V * \hat{H}_i + g'\frac{Y}{2}\overline{\hat{H}}_i * \hat{D}_{V'} * \hat{H}_i - \\ & - (\sqrt{2}g\overline{\hat{h}}_{i\,\alpha} * \overline{\hat{\lambda}}^\alpha * \hat{H}_i + \sqrt{2}g'\frac{Y}{2}\overline{\hat{h}_{i\alpha}} * \overline{\hat{\lambda}'}^{\alpha\dot{\alpha}} * H_i + c.c) \Big]. \end{aligned} \tag{36}$$

The noncommutative supersymmetric leptonic matter sector is:

$$\begin{aligned} S_{Matter,leptonic} = & \int d^4x \Big[\overline{\hat{D}_\mu \hat{L}} * \hat{D}^\mu \hat{L} + i\hat{l}^\alpha * \sigma^\mu_{\alpha\dot{\alpha}} \overline{\hat{D}_\mu \hat{l}}^{\alpha\dot{\alpha}} + \overline{\hat{F}_L} * \hat{F}_L + g\overline{\hat{L}} * \hat{D}_V * \hat{L} + \\ & + g'\frac{Y}{2}\overline{\hat{L}} * \hat{D}_{V'} * \hat{L} - \left(\sqrt{2}g\overline{\hat{l}_{\dot{\alpha}}} * \overline{\hat{\lambda}^\alpha}^{\dot{\alpha}} * \hat{L} + \sqrt{2}g'\frac{Y}{2}\overline{\hat{l}}_{\dot{\alpha}} * \overline{\hat{\lambda}'}^{\dot{\alpha}} * \hat{L} + c.c \right) \Big] + \\ & + \int d^4x \Big[\overline{\hat{D}_\mu \hat{R}} * \hat{D}^\mu \hat{R} + i\hat{r}^\alpha * \sigma^\mu_{\alpha\dot{\alpha}} \overline{\hat{D}_\mu \hat{r}}^{\alpha\dot{\alpha}} + \overline{\hat{F}_R} * \hat{F}_R + g'\frac{Y}{2}\overline{\hat{R}} * \hat{D}_{V'} * \hat{R} - \\ & - \left(\sqrt{2}g'\frac{Y}{2}\overline{\hat{r}}_{\dot{\alpha}} * \overline{\hat{\lambda}'}^{\dot{\alpha}} * \hat{R} + c.c \right) \Big]. \end{aligned} \tag{37}$$

And the quark sector is:

$$
\begin{aligned}
S_{Matter,quark} = &\int d^4x(\overline{\hat{D}_\mu \hat{Q}} * \hat{D}^\mu \hat{Q} + i\hat{q}^\alpha * \sigma^\mu_{\alpha\dot{\alpha}} \overline{\hat{D}_\mu \hat{q}}^{\alpha\dot{\alpha}} + \overline{\hat{F}_Q} * \hat{F}_Q + g\overline{\hat{Q}} * \hat{D}_V * \hat{Q} + \\
&+ g'\frac{Y}{2}\overline{\hat{Q}} * \hat{D}_{V'} * \hat{Q} - (\sqrt{2}g\overline{\hat{q}}_{\dot{\alpha}} * \overline{\hat{\lambda}}^{\dot{\alpha}} * \hat{Q} + \sqrt{2}g'\frac{Y}{2}\overline{\hat{q}}_{\dot{\alpha}} * \overline{\hat{\lambda}'}^{\dot{\alpha}} * \hat{Q} + c.c) + \\
&+ \int d^4x\Big[\overline{\hat{D}_\mu \hat{U}} * \hat{D}^\mu \hat{U} + i\hat{u}^\alpha * \sigma^\mu_{\alpha\dot{\alpha}} \overline{\hat{D}_\mu \hat{u}}^{\alpha\dot{\alpha}} + \overline{\hat{F}_U} * \hat{F}_U + g'\frac{Y}{2}\overline{\hat{U}} * \hat{D}_{V'} * \hat{U} - \\
&- (\sqrt{2}g'\frac{Y}{2}\overline{\hat{u}_{\dot{\alpha}}} * \overline{\lambda^{,}}^{\dot{\alpha}} * \hat{U} + c.c)\Big] + \\
&+ \int d^4x\Big[\overline{\hat{D}_\mu \hat{D}} * \hat{D}^\mu \hat{D} + i\hat{d}^\alpha * \sigma^\mu_{\alpha\dot{\alpha}} \overline{\hat{D}_\mu \hat{d}}^{\alpha\dot{\alpha}} + \overline{\hat{F}_D} * \hat{F}_D + \\
&+ g'\frac{Y}{2}\overline{\hat{D}} * \hat{D}_{V'} * \hat{D} - (\sqrt{2}g'\frac{Y}{2}\overline{d}_{\dot{\alpha}} * \overline{\lambda^{,}}^{\dot{\alpha}} * \hat{D} + c.c)\Big]
\end{aligned}
\tag{38}
$$

For the noncommutative supersymmetric gauge sector of the action, one gets:

$$
\begin{aligned}
S_{Gauge} = &\int d^4x 2tr\left(-\frac{1}{4}\hat{F}_{\mu\nu} * \hat{F}^{\mu\nu} + i\hat{\lambda}^\alpha * \sigma^\mu_{\alpha\dot{\alpha}} \overline{\hat{D}_\mu \hat{\lambda}}^{\alpha\dot{\alpha}} + \frac{1}{2}\hat{D}_V * \hat{D}_V\right) \\
&+ \int d^4x\left(-\frac{1}{4}\hat{F}'_{\mu\nu} * F^{,\mu\nu} + i\hat{\lambda}'^\alpha * \sigma^\mu_{\alpha\dot{\alpha}} \overline{\hat{D}_\mu \lambda^{,}}^{\alpha\dot{\alpha}} + \frac{1}{2}\hat{D}_{V'} * \hat{D}_{V'}\right)
\end{aligned}
\tag{39}
$$

The last term is the noncommutative supersymmetric interactions term:

$$
\begin{aligned}
S_{interaction} = &f_R \int d^4x\Big[-\hat{H}_1^t(i\sigma_2) * \hat{l} * \hat{r} + \hat{H}_1^t(i\sigma_2) * \hat{F}_R * \hat{R} + \hat{H}_1^t(i\sigma_2) * \hat{L} * \hat{F}_R - \\
&-\hat{h}_1^{t\alpha}(i\sigma_2) * \hat{l}_\alpha * \hat{R} - \hat{h}_1^{t\alpha}(i\sigma_2) * \hat{L} * r_\alpha + \hat{F}_1^t(i\sigma_2) * \hat{L} * \hat{R}\Big] + \\
&+f_U \int d^4x\Big[-\hat{H}_2^t(i\sigma_2) * \hat{q} * \hat{u} + \hat{H}_2^t(i\sigma_2) * \hat{F}_U * \hat{U} + \hat{H}_2^t(i\sigma_2) * \hat{Q} * \hat{F}_U - \\
&-\hat{h}_2^{t\alpha}(i\sigma_2) * \hat{q}_\alpha * \hat{U} - \hat{h}_2^{t\alpha}(i\sigma_2) * \hat{Q} * \hat{u}_\alpha + \hat{F}_2^t(i\sigma_2) * \hat{Q} * \hat{U}\Big] + \\
&+f_D \int d^4x\Big[-\hat{H}_1^t(i\sigma_2) * \hat{q} * \hat{d} + \hat{H}_1^t * (i\sigma_2) * \hat{F}_D * \hat{D} + \hat{H}_1^t(i\sigma_2) * \hat{Q} * \hat{F}_D - \\
&-\hat{h}_1^{t\alpha}(i\sigma_2) * \hat{q}_\alpha * \hat{D} - \hat{h}_1^{t\alpha}(i\sigma_2) * \hat{Q} * \hat{d}_\alpha + \hat{F}_1^t(i\sigma_2) * \hat{Q} * \hat{D}\Big] + \\
&+\mu \int d^4x[-\hat{H}_1^t(i\sigma_2) * \hat{F}_2 + \hat{F}_1^t(i\sigma_2) * \hat{H}_2 + \hat{h}_1^{t\alpha}(i\sigma_2) * \hat{h}_{2\alpha}] + c.c
\end{aligned}
\tag{40}
$$

To give the expression of this action in terms of the ordinary fields, we have to write the noncommutative fields in terms of the ordinary ones at the first order in $\theta^{\mu\nu}$, and then to replace the ordinary product by the moyal star product . We start here with the first step:

$$
\hat{V}_\mu = V_\mu + \frac{1}{4}\theta^{\rho\sigma}\left[V_\sigma \partial_\rho V_\mu + \partial_\rho V V_\sigma + V_\sigma F_{\rho\mu} + F_{\rho\mu} V_\sigma\right], \tag{41}
$$

$$\hat{V}'_\mu = V'_\mu + \frac{1}{2}\theta^{\rho\sigma}\left[V'_\sigma\partial_\rho V'_\mu + V'_\sigma F'_{\rho\mu}\right] \tag{42}$$

$$\hat{\lambda}^\alpha = \lambda^\alpha + \frac{1}{2}\theta^{\rho\sigma}V_\sigma\partial_\rho\lambda^\alpha + \frac{i}{4}\theta^{\rho\sigma}V_\rho V_\sigma\lambda^\alpha \tag{43}$$

$$\hat{\lambda}'^\alpha = \lambda'^\alpha - \theta^{\rho\sigma}V'_\rho\partial_\sigma\lambda'^\alpha \tag{44}$$

$$\hat{l}^\alpha = l^\alpha - \frac{1}{2}\theta^{\rho\sigma}\left(V_\rho + V'_\rho\right)\partial_\sigma l^\alpha + \frac{i}{4}\theta^{\rho\sigma}\left(V_\rho + V'_\rho\right)\left(V_\sigma + V'_\sigma\right)l^\alpha \tag{45}$$

$$\hat{r}^\alpha = r^\alpha - \frac{1}{2}\theta^{\rho\sigma}V'_\rho\partial_\sigma r^\alpha \tag{46}$$

$$\hat{h}_i^\alpha = h_i^\alpha - \frac{1}{2}\theta^{\rho\sigma}\left(V_\rho + V'_\rho\right)\partial_\sigma h_i^\alpha + \frac{i}{4}\theta^{\rho\sigma}\left(V_\rho + V'_\rho\right)\left(V_\sigma + V'_\sigma\right)h_i^\alpha \tag{47}$$

$$\hat{L} = L - \frac{1}{2}\theta^{\rho\sigma}\left(V_\rho + V'_\rho\right)\partial_\sigma L + \frac{i}{4}\theta^{\rho\sigma}\left(V_\rho + V'_\rho\right)\left(V_\sigma + V'_\sigma\right)L \tag{48}$$

$$\hat{R} = R - \frac{1}{2}\theta^{\rho\sigma}V'_\rho\partial_\sigma R \tag{49}$$

$$\hat{H}_i = H_i - \frac{1}{2}\theta^{\rho\sigma}\left(V_\rho + V'_\rho\right)\partial_\sigma H_i + \frac{i}{4}\theta^{\rho\sigma}\left(V_\rho + V'_\rho\right)\left(V_\sigma + V'_\sigma\right)H_i \tag{50}$$

$$\hat{D}_V = D_V - \theta^{\rho\sigma}V_\rho\partial_\sigma D_V \tag{51}$$

$$\hat{D}_{V'} = D_{V'} - \theta^{\rho\sigma}V'_\rho\partial_\sigma D_{V'} \tag{52}$$

$$\hat{F}_L = F_L - \theta^{\rho\sigma}(V_\rho + V'_\rho)\partial_\sigma F_L \tag{53}$$

$$\hat{F}_R = F_R - \theta^{\rho\sigma}V'_\rho\partial_\sigma F_R \tag{54}$$

$$\hat{F}_i = F_i - \theta^{\rho\sigma}(V_\rho + V'_\rho)\partial_\sigma F_i \tag{55}$$

Next, we write the noncommutative supersymmetric transformations as the ordinary ones but with replacing the fields by their partner via Seiberg-Witten map and we use the fermionic constant spinor parameter ε:

$$\delta\ \hat{V}_\mu = i\varepsilon^\alpha\sigma_{\mu\alpha\dot{\alpha}}\overline{\hat{\lambda}}^{\dot{\alpha}} + i\overline{\varepsilon}^\alpha\overline{\sigma}_{\alpha\dot{\alpha}\mu}\hat{\lambda}^\alpha \tag{56}$$

$$\delta\ \hat{V}'_\mu = i\varepsilon^\alpha\sigma_{\alpha\dot{\alpha}\mu}\overline{\hat{\lambda}'}^{\alpha\dot{\alpha}} + i\overline{\varepsilon}^{\dot{\alpha}}\overline{\sigma}_{\mu\dot{\alpha}\alpha}\hat{\lambda}'^{\dot{\alpha}} \tag{57}$$

$$\delta\hat{\lambda}^\alpha = i\varepsilon^\alpha\hat{D}_V + i\sigma_\beta^{\alpha\mu\nu}\varepsilon^\beta\hat{F}_{\mu\nu} \tag{58}$$

$$\delta\hat{\lambda}'^\alpha = i\varepsilon^\alpha\hat{D}_{V'} + i\sigma_\beta^{\alpha\mu\nu}\varepsilon^\beta\hat{F}'_{\mu\nu} \tag{59}$$

$$\delta\hat{l}^{\alpha} = \sqrt{2}\varepsilon^{\alpha}\hat{F}_L + i\sqrt{2}\sigma^{\alpha\mu}_{\beta}\bar{\varepsilon}^{\beta}\hat{D}_{\mu} * \hat{L} \quad (60)$$

$$\delta\hat{r}^{\alpha} = \sqrt{2}\varepsilon^{\alpha}\hat{F}_R + i\sqrt{2}\sigma^{\alpha\mu}_{\beta}\bar{\varepsilon}^{\beta}\hat{D}_{\mu} * \hat{R} \quad (61)$$

$$\delta\hat{h}_i^{\alpha} = \sqrt{2}\varepsilon^{\alpha}\hat{F}_i + i\sqrt{2}\sigma^{\alpha\mu}_{\beta}\bar{\varepsilon}^{\beta}\hat{D}_{\mu} * \hat{H}_i \quad (62)$$

$$\delta\hat{L} = \sqrt{2}\varepsilon^{\alpha}\hat{l}^{\alpha} \quad (63)$$

$$\delta\hat{H}_i = \sqrt{2}\varepsilon^{\alpha}\hat{h}_i^{\alpha} \quad (64)$$

$$\delta\hat{D}_V = \bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\dot{\alpha}\beta}\hat{D}_{\mu} * \hat{\lambda}^{\beta} - \varepsilon^{\alpha}\sigma^{\mu}_{\alpha\beta}\hat{D}_{\mu} * \bar{\hat{\lambda}}^{\beta} \quad (65)$$

$$\delta\hat{D}_{V'} = \bar{\varepsilon}^{\alpha}\bar{\sigma}^{\mu}_{\alpha\dot{\alpha}}\hat{D}_{\mu} * \hat{\lambda}'^{\dot{\alpha}} - \varepsilon^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\hat{D}_{\mu} * \bar{\hat{l}}'^{\dot{\alpha}} \quad (66)$$

$$\delta\hat{F}_L = i\sqrt{2}\bar{\varepsilon}^{\alpha}\bar{\sigma}^{\mu}_{\alpha\dot{\alpha}}\hat{D}_{\mu} * \hat{l}^{\dot{\alpha}} - 2g\bar{\varepsilon}^{\alpha}\bar{\hat{\lambda}}_{\alpha} * \hat{L} - g'Y\bar{\varepsilon}^{\alpha}\bar{\hat{\lambda}}'_{\alpha} * \hat{L} \quad (67)$$

$$\delta\hat{F}_R = i\sqrt{2}\bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha}\hat{D}_{\mu} * \hat{r}^{\dot{\alpha}} - g'Y\bar{\varepsilon}^{\dot{\alpha}}\bar{\hat{\lambda}}'_{\dot{\alpha}} * \hat{R} \quad (68)$$

$$\delta\hat{F}_i = i\sqrt{2}\bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\alpha\dot{\alpha}}\hat{D}_{\mu} * \hat{h}_i^{\dot{\alpha}} - 2g\bar{\varepsilon}^{\dot{\alpha}}\bar{\hat{\lambda}}_{\dot{\alpha}} * \hat{H}_i - g'Y\bar{\varepsilon}^{\dot{\alpha}}\bar{\hat{\lambda}}'_{\dot{\alpha}} * \hat{H}_i \quad (69)$$

With this definition we give the final form for each term in the total NCMSSM action:

$$S_{NCMSSM} = S_{MSSM} + S^1 \quad (70)$$

We write it on this form to show that it is just an extension of the classical action with S^1 the first order in θ part. It takes the form:

$$S^1 = S^1{}_{Higgs} + S^1{}_{Lepton} + S^1{}_{Quark} + S^1{}_{Gauge} + S^1{}_{interaction} \quad (71)$$

The first term is the Higgs sector and it is giving by the following:

$$S^1{}_{Higgs} = \sum_i \int d^4x \Big[\overline{D_{\mu}H}_i \left(D^{\mu}H_i^1 - \frac{1}{2}\theta^{\rho\sigma}\partial_{\rho}V^{\mu}\partial_{\sigma}H_i - V_{\mu}{}^1 H_i \right) +$$

$$+ \left(D^{\mu}H_i^1 - \frac{1}{2}\theta^{\rho\sigma}\partial_{\rho}V^{\mu}\partial_{\sigma}H_i - V_{\mu}{}^1 H_i \right)^{+} D^{\mu}H_i +$$

$$+ \theta^{\rho\sigma}\Big[-\frac{i}{4}h_i^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\Gamma_{\rho\sigma}\overline{D_{\mu}h}_i^{\dot{\alpha}} - \frac{i}{2}h^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\Gamma_{\mu\rho}\overline{D_{\sigma}h}_i^{\dot{\alpha}} + \frac{1}{2}\overline{F}_i F_i F'_{\rho\sigma} + \overline{F}_i F_i \partial_{\rho}V_{\sigma} +$$

$$+ \frac{g}{4}\overline{H}_i\Gamma_{\sigma\rho}D_V H_i + \frac{g}{2}\overline{H}_i\partial_{\sigma}D_V W_{\rho}H_i + g\overline{H}_i\partial_{\sigma}D_V V_{\rho}H_i + \frac{i}{2}\overline{H}_i\partial_{\rho}D_V\partial_{\sigma}H_i +$$

$$+ \frac{g'}{8}Y\overline{H}_i\Gamma_{\sigma\rho}D_V H_i + \frac{g'}{4}Y\overline{H}_i\partial_{\sigma}D_V W_{\rho}H_i + g'\frac{Y}{2}\overline{H}_i\partial_{\sigma}D_V V'_{\rho}H_i +$$

$$+\frac{ig'}{4}Y\overline{H}_i\partial_\rho D_V V_\sigma H_i-\Big(\frac{1}{4}\sqrt{2}g\overline{h}_i^{\dot\alpha}\overline{\lambda}_{\dot\alpha}\Gamma_{\sigma\rho}H_i+\sqrt{2}g\overline{h}_i^{\dot\alpha}\overline{\partial_\sigma\lambda}_{\dot\alpha}V_\rho H_i+$$
$$+\frac{i}{4}\sqrt{2}g\overline{h}_{i\dot\alpha}\overline{\lambda}^{\dot\alpha}V_\rho V_\sigma H_i+\frac{i}{2}\sqrt{2}g\overline{h}_{i\dot\alpha}\overline{\partial_\rho\lambda}^{\dot\alpha}\partial_\sigma H_i+\frac{1}{8}\sqrt{2}g'Y\overline{h}_{i\dot\alpha}\overline{\lambda'}^{\dot\alpha}\Gamma_{\sigma\rho}H_i+ \tag{72}$$
$$+\frac{1}{2}\sqrt{2}g'Y\overline{h}_{i\dot\alpha}\overline{\partial_\sigma\lambda'}^{\dot\alpha}W_\rho H_i+\frac{i}{4}\sqrt{2}g'Y\overline{h}_{i\dot\alpha}\overline{\partial_\rho\lambda'}^{\dot\alpha}\partial_\sigma H_i+c.c\Big)\Big]\Big],$$

Where we use the notations:

$$\Gamma_{\sigma\rho}=F_{\sigma\rho}+F'_{\sigma\rho}\ \text{ and }\ W_\rho=V_\rho+V'_\rho$$

The second term is the leptonic matter one and we subdivide it in two parts. One for the left handed leptons and the other for the right handed leptons. We have to note that the two parts differs because the left handed leptons gather in doublets when the right handed ones do it in singlets. So the leptonic matter term takes the form:

$$S^1_{Leptonic}=S^1_{L-h,lepton}+S^1_{R-h,lepton}, \tag{73}$$

Where the left and right handed parts write:

$$S^1_{L-h,lepton}=\int d^4x\Big[\overline{D_\mu L}(D^\mu L^1-\frac{1}{2}\theta^{\rho\sigma}\partial_\rho V^\mu\partial_\sigma L-V_\mu{}^1L)+$$
$$+(D^\mu L^1-\frac{1}{2}\theta^{\rho\sigma}\partial_\rho V^\mu\partial_\sigma L-V_\mu{}^1L)^+D^\mu L+$$
$$+\theta^{\rho\sigma}\Big[-\frac{i}{4}l^\alpha\sigma^\mu_{\alpha\dot\alpha}\Gamma_{\rho\sigma}\overline{D_\mu l}^{\dot\alpha}-\frac{i}{2}l^\alpha\sigma^\mu_{\alpha\dot\alpha}\Gamma_{\mu\rho}\overline{D_\sigma l}^{\dot\alpha}+\frac{1}{2}\overline{F_L}FF'_{\rho\sigma}+\overline{F_L}F\partial_\rho V_\sigma+$$
$$+\frac{g}{4}\overline{L}\Gamma_{\sigma\rho}D_V L+\frac{g}{2}\overline{L}\partial_\sigma D_V W_\rho L+g\overline{L}\partial_\sigma D_V V_\rho L+\frac{i}{2}\overline{L}\partial_\rho D_V V_\sigma H_i+$$
$$+\frac{g'}{8}Y\overline{L}\Gamma_{\sigma\rho}D_V H_i+\frac{g'}{4}Y\overline{L}\partial_\sigma D_V W_\rho L+g'\frac{Y}{2}\overline{L}\partial_\sigma D_V V'_\rho L+\frac{ig'}{4}Y\overline{L}\partial_\rho D_V V_\sigma L- \tag{74}$$
$$-\Big(\frac{1}{4}\sqrt{2}g\bar{l}_{\dot\alpha}\overline{\lambda}^{\dot\alpha}\Gamma_{\sigma\rho}L+\sqrt{2}g\bar{l}_{\dot\alpha}\overline{\partial_\sigma\lambda}^{\dot\alpha}V_\rho L+\frac{i}{4}\sqrt{2}g\bar{l}_{\dot\alpha}\overline{\lambda}^{\dot\alpha}V_\rho V_\sigma L+$$
$$+\frac{i}{2}\sqrt{2}g\bar{l}_{\dot\alpha}\overline{\partial_\rho\lambda}^{\dot\alpha}\partial_\sigma L+\frac{1}{8}\sqrt{2}g'Y\bar{l}_{\dot\alpha}\overline{\lambda'}^{\dot\alpha}\Gamma_{\sigma\rho}L+\frac{1}{2}\sqrt{2}g'Y\bar{l}_{\dot\alpha}\overline{\partial_\sigma\lambda'}^{\dot\alpha}W_\rho L+$$
$$+\frac{i}{4}\sqrt{2}g'Y\bar{l}_{\dot\alpha}\overline{\partial_\rho\lambda'}^{\dot\alpha}\partial_\sigma L+c.c\Big)\Big]\Big],$$

$$S^1_{R-h,lepton}=\int d^4x\theta^{\rho\sigma}\Big[-\frac{1}{2}\overline{D_\mu R}D_\mu V_\sigma\partial_\rho R+\frac{i}{2}\overline{D_\mu R}\partial_\mu V'^\mu V'_\rho R+$$
$$+\frac{i}{2}\overline{D_\mu R}F'^\mu_\sigma V'_\rho R-\frac{1}{4}\sigma^\mu_{\alpha\dot\alpha}\overline{r}^\alpha F'_{\rho\sigma}D_\mu r^\alpha-\frac{1}{2}\sigma^\mu_{\alpha\dot\alpha}\overline{r}^{\dot\alpha}F'_{\rho\sigma}D_\sigma r^\alpha+$$
$$+\overline{F}_R F_R\partial_\sigma V'_\rho+\frac{1}{4}g'\frac{Y}{2}\overline{R}D_{V'}RF'_{\sigma\rho}+\frac{1}{2}g'\frac{Y}{2}\overline{R}\partial_\rho D_{V'}RV'_\sigma-$$

$$-\Big(\frac{1}{4}\sqrt{2}g'\frac{Y}{2}\overline{r}^{\dot{\alpha}}\overline{\lambda'}_{\dot{\alpha}}RF'_{\sigma\rho}+\sqrt{2}g'\frac{Y}{2}\overline{r}^{\dot{\alpha}}\partial_{\rho}\overline{\lambda}_{\dot{\alpha}}RV'_{\sigma}+$$
$$+\frac{i}{2}\sqrt{2}g'\frac{Y}{2}\overline{r}^{\dot{\alpha}}\partial_{\rho}\overline{\lambda'}_{\dot{\alpha}}D'_{\sigma}R+c.c\Big)\Big]. \tag{75}$$

For the quark sector, we use the same convention as for the lepton part and we make the same remarks for differences between left handed and right handed terms. So it is given by the expression:

$$S^{1}{}_{Quark}=S^{1}{}_{L-h,quark}+S^{1}{}_{R-h,quark}. \tag{76}$$

First, the left-handed quarks term takes the form:

$$S^{1}_{L-h,quark}=\int d^{4}x\Big[\overline{D_{\mu}Q}(D^{\mu}Q^{1}-\frac{1}{2}\theta^{\rho\sigma}\partial_{\rho}V^{\mu}\partial_{\sigma}Q-V_{\mu}{}^{1}Q)+$$
$$+\Big(D^{\mu}Q^{1}-\frac{1}{2}\theta^{\rho\sigma}\partial_{\rho}V^{\mu}\partial_{\sigma}Q-V_{\mu}{}^{1}Q\Big)^{+}D^{\mu}Q+\theta^{\rho\sigma}\Big[-\frac{i}{4}q^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\Gamma_{\rho\sigma}\overline{D_{\mu}q}^{\dot{\alpha}}-$$
$$-\frac{i}{2}q^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\Gamma_{\mu\rho}\overline{D_{\sigma}q}^{\dot{\alpha}}+\frac{1}{2}\overline{F}_{Q}F_{Q}F'_{\rho\sigma}+\overline{F}_{Q}F_{Q}\partial_{\rho}V_{\sigma}+\frac{g}{4}\overline{L}\Gamma_{\sigma\rho}D_{V}L+$$
$$+\frac{g}{2}\overline{L}\partial_{\sigma}D_{V}W_{\rho}L+g\overline{L}\partial_{\sigma}D_{V}V_{\rho}L+\frac{i}{2}\overline{L}\partial_{\rho}D_{V}V_{\sigma}H_{i}+\frac{g'}{8}Y\overline{Q}\Gamma_{\sigma\rho}D_{V}Q+$$
$$+\frac{g'}{4}Y\overline{Q}\partial_{\sigma}D_{V}W_{\rho}Q+g'\frac{Y}{2}\overline{Q}\partial_{\sigma}D_{V}V'_{\rho}Q+\frac{ig'}{4}Y\overline{Q}\partial_{\rho}D_{V}V_{\sigma}Q-$$
$$-\Big(\frac{1}{4}\sqrt{2}g\overline{q}_{\dot{\alpha}}\overline{\lambda}^{\dot{\alpha}}\Gamma_{\sigma\rho}Q+\sqrt{2}g\overline{q}_{\dot{\alpha}}\overline{\partial_{\sigma}\lambda}^{\dot{\alpha}}V_{\rho}Q+\frac{i}{4}\sqrt{2}g\overline{q}_{\dot{\alpha}}\overline{\lambda}^{\dot{\alpha}}V_{\rho}V_{\sigma}Q+$$
$$+\frac{i}{2}\sqrt{2}g\overline{q}_{\dot{\alpha}}\overline{\partial_{\rho}\lambda}^{\dot{\alpha}}\partial_{\sigma}Q+\frac{1}{8}\sqrt{2}g'Y\overline{q}_{\dot{\alpha}}\overline{\lambda'}^{\dot{\alpha}}\Gamma_{\sigma\rho}Q+\frac{1}{2}\sqrt{2}g'Y\overline{q}_{\dot{\alpha}}\overline{\partial_{\sigma}\lambda'}^{\dot{\alpha}}W_{\rho}Q+$$
$$+\frac{i}{4}\sqrt{2}g'Y\overline{q}_{\dot{\alpha}}\overline{\partial_{\rho}\lambda'}^{\dot{\alpha}}\partial_{\sigma}Q+c.c\Big)\Big]\Big]. \tag{77}$$

For right-handed quarks, the expression is:

$$S^{1}_{R-h,quark}=\int d^{4}x\theta^{\rho\sigma}\Big[-\frac{1}{2}\overline{D_{\mu}U}D_{\mu}V_{\sigma}\partial_{\rho}U+\frac{i}{2}\overline{D_{\mu}U}\partial_{\mu}V'^{\mu}V'_{\rho}U+$$
$$+\frac{i}{2}\overline{D_{\mu}U}F'^{\mu}_{\sigma}V'_{\rho}U-\frac{1}{4}\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{u}^{\dot{\alpha}}F'_{\rho\sigma}D_{\mu}u^{\alpha}-\frac{1}{2}\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{u}^{\dot{\alpha}}F'_{\rho\sigma}D_{\sigma}u^{\dot{\alpha}}+$$
$$+\overline{F_{U}}F_{U}\partial_{\sigma}V'_{\rho}+\frac{1}{4}g'\frac{Y}{2}\overline{U}D_{V'}UF'_{\sigma\rho}+\frac{1}{2}g'\frac{Y}{2}\overline{U}\partial_{\rho}D_{V'}UV'_{\sigma}-$$
$$-\Big(\frac{1}{4}\sqrt{2}g'\frac{Y}{2}\overline{u}^{\dot{\alpha}}\overline{\lambda'}_{\dot{\alpha}}UF'_{\sigma\rho}+\sqrt{2}g'\frac{Y}{2}\overline{u}^{\dot{\alpha}}\partial_{\rho}\overline{\lambda'}_{\dot{\alpha}}UV'_{\sigma}+$$
$$+\frac{i}{2}\sqrt{2}g'\frac{Y}{2}\overline{u}^{\dot{\alpha}}\partial_{\rho}\overline{\lambda'}_{\dot{\alpha}}D'_{\sigma}U+c.c\Big)-\frac{1}{2}\overline{D_{\mu}D}D_{\mu}V_{\sigma}\partial_{\rho}D+$$
$$+\frac{i}{2}\overline{D_{\mu}D}\partial_{\mu}V'^{\mu}V'_{\rho}D+\frac{i}{2}\overline{D_{\mu}D}F'^{\mu}_{\sigma}V'_{\rho}D-\frac{1}{4}\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{d}^{\dot{\alpha}}F'_{\rho\sigma}D_{\mu}d^{\alpha}-$$

$$-\frac{1}{2}\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{d}^{\dot{\alpha}}F'_{\rho\sigma}D_{\sigma}d^{\alpha}+\overline{F_D}F_D\partial_{\sigma}V'_{\rho}+\frac{1}{4}g'\frac{Y}{2}\overline{D}D_{V'}DF'_{\sigma\rho}+$$
$$+\frac{1}{2}g'\frac{Y}{2}\overline{D}\partial_{\rho}D_{V'}DV'_{\sigma}-\left(\frac{1}{4}\sqrt{2}g'\frac{Y}{2}\overline{d}^{\dot{\alpha}}\overline{\lambda'}_{\dot{\alpha}}DF'_{\sigma\rho}+\right. \tag{78}$$
$$\left.\left.+\sqrt{2}g'\frac{Y}{2}\overline{d}^{\dot{\alpha}}\partial_{\rho}\overline{\lambda'}_{\dot{\alpha}}DV'_{\sigma}+\frac{i}{2}\sqrt{2}g'\frac{Y}{2}\overline{d}^{\dot{\alpha}}\partial_{\rho}\overline{\lambda'}_{\dot{\alpha}}D'_{\sigma}D+c.c\right)\right].$$

For the gauge sector, one finds:

$$S^{1}_{Gauge}=\int d^4x\,2tr\theta^{\mu\nu}\left(F_{\nu\rho}F^{\rho\sigma}F_{\sigma\mu}-\frac{1}{4}F_{\nu\mu}F_{\rho\sigma}F^{\sigma\rho}+\frac{1}{4}\lambda\sigma^{\rho}\partial_{\rho}\overline{\lambda}F_{\mu\nu}+\right.$$
$$\left.\left.+\lambda\sigma^{\rho}\partial_{\mu}\overline{\lambda}F_{\nu\rho}\right)+\frac{1}{4}D^2{}_V F_{\mu\nu}\right)+ \tag{79}$$
$$+\int d^4x\theta^{\mu\nu}\left(\frac{1}{2}\lambda'\sigma^{\rho}\partial_{\rho}\overline{\lambda'}F_{\mu\nu}+\lambda'\sigma^{\rho}\partial_{\mu}\overline{\lambda'}F_{\nu\rho}+\frac{1}{2}D^2_{V'}F'_{\mu\nu}\right)$$

The interaction term takes the form:

$$S^{1}_{interaction}=S^{1}_{H.L}+S^{1}_{H.q}+S^{1}_{H.H} \tag{80}$$

Where the first part refers to the coupling between Higgs and leptons:

$$S^{1}_{H.L}=f_R\int d^4x\theta^{\rho\sigma}\left[-\frac{1}{4}H_1^t(i\sigma_2)lr\Gamma_{\sigma\rho}-\frac{1}{2}H_1^t(i\sigma_2)V_{\rho}l\partial_{\sigma}r-\right.$$
$$-\frac{i}{2}H_1^t(i\sigma_2)\partial_{\rho}l\partial_{\sigma}r+\frac{1}{4}H_1^t(i\sigma_2)F_LR\Gamma_{\sigma\rho}-\frac{1}{2}H_1^t(i\sigma_2)W_{\rho}\partial_{\sigma}F_LR+$$
$$+\frac{1}{2}H_1^t(i\sigma_2)V_{\rho}F_L\partial_{\sigma}R+\frac{i}{2}H_1^t(i\sigma_2)\partial_{\rho}F_L\partial_{\sigma}R+\frac{i}{8}H_1^t(i\sigma_2)W_{\rho}W_{\sigma}F_LR+$$
$$+\frac{1}{4}H_1^t(i\sigma_2)LF_R\Gamma_{\sigma\rho}-\frac{1}{2}H_1^t(i\sigma_2)V'_{\rho}L\partial_{\sigma}F_R+\frac{1}{2}H_1^t(i\sigma_2)V_{\rho}L\partial_{\sigma}F_R+$$
$$+\frac{i}{2}H_1^t(i\sigma_2)\partial_{\rho}L\partial_{\sigma}F_R-\frac{1}{4}h_1^t(i\sigma_2)lR\Gamma_{\sigma\rho}-\frac{1}{2}h_1^t(i\sigma_2)V_{\rho}l\partial_{\sigma}R- \tag{81}$$
$$+\frac{i}{2}h_1^t(i\sigma_2)\partial_{\rho}l\partial_{\sigma}R-\frac{1}{4}h_1^t(i\sigma_2)Lr\Gamma_{\sigma\rho}-\frac{1}{2}h_1^t(i\sigma_2)V_{\rho}L\partial_{\sigma}r-$$
$$-\frac{i}{2}h_1^t(i\sigma_2)\partial_{\rho}L\partial_{\sigma}r+\frac{1}{4}F_1^t(i\sigma_2)LR\Gamma_{\sigma\rho}-\frac{1}{2}\partial_{\sigma}F_1^t(i\sigma_2)W_{\rho}LR+$$
$$\left.+\frac{1}{2}F_1^t(i\sigma_2)V_{\rho}L\partial_{\sigma}R+\frac{i}{2}F_1^t(i\sigma_2)\partial_{\rho}L\partial_{\sigma}R+\frac{i}{8}F_1^t(i\sigma_2)W_{\rho}W_{\sigma}LR+c.c\right].$$

The second part expresses the coupling between Higgs and quarks:

$$S^{1}_{H.q}=f_U\int d^4x\theta^{\rho\sigma}\left[-\frac{1}{4}H_2^t(i\sigma_2)qu\Gamma_{\sigma\rho}-\frac{1}{2}H_2^t(i\sigma_2)V_{\rho}q\partial_{\sigma}u-\right.$$
$$-\frac{i}{2}H_2^t(i\sigma_2)\partial_{\rho}q\partial_{\sigma}u+\frac{1}{4}H_2^t(i\sigma_2)F_QU\Gamma_{\sigma\rho}-\frac{1}{2}H_2^t(i\sigma_2)W_{\rho}\partial_{\sigma}F_QU+$$

$$
\begin{aligned}
&+\frac{1}{2}H_2^t(i\sigma_2)V_\rho F_Q\partial_\sigma U+\frac{i}{2}H_2^t(i\sigma_2)\partial_\rho F_Q\partial_\sigma U+\frac{i}{8}H_2^t(i\sigma_2)W_\rho W_\sigma F_Q U+\\
&+\frac{1}{4}H_2^t(i\sigma_2)QF_U\Gamma_{\sigma\rho}-\frac{1}{2}H_2^t(i\sigma_2)V'_\rho Q\partial_\sigma F_U+\frac{1}{2}H_2^t(i\sigma_2)V_\rho Q\partial_\sigma F_U+\\
&+\frac{i}{2}H_2^t(i\sigma_2)\partial_\rho Q\partial_\sigma F_U-\frac{1}{4}h_2^t(i\sigma_2)QU\Gamma_{\sigma\rho}-\frac{1}{4}h_2^t(i\sigma_2)qU\Gamma_{\sigma\rho}-\\
&-\frac{i}{2}h_2^t(i\sigma_2)V_\rho q\partial_\sigma U-\frac{1}{4}h_2^t(i\sigma_2)\partial_\rho q\partial_\sigma U-\frac{1}{4}h_2^t(i\sigma_2)QU\Gamma_{\sigma\rho}-\\
&-\frac{1}{2}h_2^t(i\sigma_2)V_\rho Q\partial_\sigma u-\frac{i}{2}h_2^t(i\sigma_2)\partial_\rho Q\partial_\sigma u+\frac{1}{4}F_2^t(i\sigma_2)QU\Gamma_{\sigma\rho}-\\
&-\frac{1}{2}F_2^t(i\sigma_2)W_\rho QU+\frac{1}{2}F_2^t(i\sigma_2)V_\rho Q\partial_\sigma U+\frac{i}{2}F_2^t(i\sigma_2)\partial_\rho Q\partial_\sigma U+\\
&+\frac{i}{8}F_2^t(i\sigma_2)W_\rho W_\sigma QU+c.c\Big].
\end{aligned}
\tag{82}
$$

And finally, the pure Higgs coupling writes:

$$
\begin{aligned}
S_{H.H}^1=\mu\int d^4x\theta^{\rho\sigma}[&-\frac{1}{4}H_1^t(i\sigma_2)F_2\Gamma_{\sigma\rho}-\frac{i}{2}D_\rho H_1^t(i\sigma_2)\partial_\sigma F_2\\
&-\frac{i}{8}H_1^t(i\sigma_2)W_\rho W_\sigma F_2-\frac{1}{4}F_1^t(i\sigma_2)H_2\Gamma_{\sigma\rho}-\frac{i}{2}\partial_\rho H_1^t(i\sigma_2)\partial_\sigma F_2\\
&+\frac{i}{2}\partial_\rho F_1^t(i\sigma_2)W_\rho H_2+\frac{1}{4}h_1^t(i\sigma_2)h_2\Gamma_{\sigma\rho}+\frac{i}{2}\partial_\rho h_1^t(i\sigma_2)\partial_\sigma F_2+c.c].
\end{aligned}
\tag{83}
$$

Obviously, when we write total action, we set the surface to zero while performing partial integrals.

Here we can give the transformations which leave S_{NCMSSM} invariant. They are obtained from the ordinary ones (58, 71) by replacing ordinary fields and product and are given by:

$$
\begin{aligned}
\delta V_\mu=&\,i\varepsilon^\alpha\sigma_{\mu\alpha\dot\alpha}\bar\lambda^{\dot\alpha}+i\overline{\varepsilon^{\dot\alpha}\sigma_{\mu\alpha\dot\alpha}}\lambda^\alpha+\\
&+\theta^{\rho\sigma}\Big[+\frac{i}{2}(\varepsilon^\alpha\sigma_{\mu\alpha\dot\alpha}\bar\lambda^{\dot\alpha}+\bar\varepsilon^{\dot\alpha}\bar\sigma_{\mu\dot\alpha\alpha}\lambda^\alpha)(\partial_\rho V_\mu+F_{\rho\mu})-\\
&-\frac{1}{2}(\varepsilon^\alpha\sigma_{\mu\dot\alpha\dot\alpha}\bar\lambda^{\dot\alpha}+\bar\varepsilon^{\dot\alpha}\bar\sigma_{\mu\dot\alpha\alpha}\lambda^\alpha)V_\rho V_\sigma+\\
&+\frac{i}{2}V_\rho(\varepsilon^\alpha\sigma_{\mu\alpha\dot\alpha}\partial_\sigma\bar\lambda^{\dot\alpha}+\bar\varepsilon^{\dot\alpha}\bar\sigma_{\mu\alpha\dot\alpha}\partial_\sigma\lambda^\alpha-\varepsilon^\alpha\sigma_{\sigma\alpha\dot\alpha}\partial_\mu\bar\lambda^{\dot\alpha}-\bar\varepsilon^{\dot\alpha}\bar\sigma_{\sigma\alpha\dot\alpha}\partial_\mu\lambda^\alpha)\Big],
\end{aligned}
\tag{84}
$$

$$
\begin{aligned}
\delta\lambda_\alpha=&\,\varepsilon^\beta\sigma^{\mu\nu}_{\beta\alpha}F_{\mu\nu}+i\varepsilon_\alpha D_V+\\
&+\theta^{\mu\nu}[\frac{i}{2}\varepsilon_\alpha V_\nu\partial_\mu D_V-\frac{i}{2}(\varepsilon^\beta\sigma_{\beta\dot\alpha\nu}\bar\lambda^{\dot\alpha}+\bar\varepsilon^{\dot\alpha}\bar\sigma_{\dot\alpha\beta\nu}\lambda^\beta)\partial_\mu\lambda_\alpha+\\
&+\frac{1}{4}\varepsilon_\alpha V_\mu V_\nu(\sigma^{\rho\sigma}F_{\rho\sigma}+D_V)+\frac{1}{2}\varepsilon_\alpha V_\rho\sigma^{\mu\nu}\partial_\sigma F_{\mu\nu}+D_V]+\sigma^{\mu\nu}\varepsilon_\alpha F^1_{\mu\nu},
\end{aligned}
\tag{85}
$$

$$
\delta D_V=\bar\varepsilon^{\dot\alpha}\bar\sigma^\mu_{\dot\alpha\beta}\partial_\mu\lambda^\beta-\varepsilon^\alpha\sigma^\mu_{\alpha\dot\beta}\partial_\mu\bar\lambda^{\dot\beta}+
$$

$$+\theta^{\rho\sigma}\Big[\frac{1}{2}(\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}^{\mu}_{\dot\alpha\beta}\partial_\rho\lambda^\beta-\varepsilon^\alpha\sigma^\mu_{\alpha\dot\beta}\partial_\rho\bar{\lambda}^{\dot\beta})\partial_\sigma V_\mu+$$
$$+\frac{i}{4}(\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}^{\mu}_{\dot\alpha\beta}\partial_\mu\lambda^\beta-\varepsilon^\alpha\sigma^\mu_{\alpha\dot\beta}\partial_\mu\bar{\lambda}^{\dot\beta})V_\sigma V_\sigma+$$
$$+\frac{1}{2}(\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}^{\mu}_{\dot\alpha\beta}\partial_\rho\lambda^\beta-\varepsilon^\alpha\sigma^\mu_{\alpha\dot\beta}\partial_\rho\bar{\lambda}^{\dot\beta})F_{\mu\sigma}+i(\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}^{\sigma}_{\dot\alpha\beta}\lambda^\beta-\varepsilon^\alpha\sigma^\sigma_{\alpha\dot\beta}\bar{\lambda}^{\dot\beta})\partial_\rho D_V+ \tag{86}$$
$$+\frac{i}{4}(\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}^{\mu}_{\dot\alpha\beta}\lambda^\beta-\varepsilon^\alpha\sigma^\mu_{\alpha\dot\beta}\bar{\lambda}^{\dot\beta})[\partial_\mu V_\rho;V_\sigma]+$$
$$+\frac{i}{4}(\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}^{\mu}_{\dot\alpha\beta}\lambda^\beta-\varepsilon^\alpha\sigma^\mu_{\alpha\dot\beta}\bar{\lambda}^{\dot\beta})\big\{[V_\mu.V_\rho].V_\sigma\big\}\Big],$$

$$\delta V'_\mu=i\varepsilon^\alpha\sigma_{\alpha\dot\beta\mu}\overline{\lambda'}^{\dot\beta}+i\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}_{\dot\alpha\beta\mu}\lambda'^\beta+$$
$$+\frac{i}{2}\theta^{\rho\sigma}\Big[(\varepsilon^\alpha\sigma_{\alpha\dot\beta\sigma}\overline{\lambda'}^{\dot\beta}+\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}_{\dot\alpha\beta\sigma}\lambda'^\beta)(\partial_\rho V'_\mu+F'_{\rho\mu})- \tag{87}$$
$$+V_\rho(\varepsilon^\alpha\sigma^\sigma{}_{\alpha\dot\beta}\partial_\sigma\bar{\lambda}^{\dot\beta}+\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}^{\sigma}_{\dot\alpha\beta}\partial_\mu\lambda^\beta)\Big],$$

$$\delta\lambda'_\alpha=\varepsilon^\beta\sigma^{\mu\nu}_{\beta\alpha}F'_{\mu\nu}+i\varepsilon D_{V'}+$$
$$+i\theta^{\mu\nu}\Big[\frac{1}{2}\varepsilon_\alpha V'_\nu\partial_\mu D_{V'}+(\varepsilon^\alpha\sigma_{\alpha\dot\beta\nu}\overline{\lambda'}^{\dot\beta}+\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}_{\dot\alpha\beta\nu}\lambda'^\beta)\partial_\mu\lambda_\alpha\Big]+ \tag{88}$$
$$+\theta^{\rho\sigma}\varepsilon_\alpha\sigma^{\mu\nu}F'_{\mu\rho}F_{\nu\sigma},$$

$$\delta D_{V'}=\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}^{\mu}_{\dot\alpha\beta}\partial_\mu\lambda'^\beta-\varepsilon^\alpha\sigma^\mu_{\alpha\dot\beta}\overline{\lambda'}^{\dot\beta}+$$
$$+\theta^{\rho\sigma}\Big[\Big(\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}^{\mu}_{\dot\alpha\beta}\partial_\rho\lambda'^\beta-\varepsilon^\alpha\sigma^\mu_{\alpha\dot\beta}\partial_\rho\overline{\lambda'}^{\dot\beta}\Big)F'_{\mu\sigma}+ \tag{89}$$
$$+i\Big(\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}^{\sigma}_{\dot\alpha\beta}\lambda'^\beta-\varepsilon^\alpha\sigma^\sigma_{\alpha\dot\beta}\overline{\lambda'}^{\dot\beta}\Big)\partial_\rho D_{V'}\Big],$$

$$\delta L=\sqrt{2}\varepsilon_\alpha l^\alpha+\frac{i}{2}\theta^{\rho\sigma}[\varepsilon^\alpha\sigma_{\alpha\dot\beta\rho}(\overline{\lambda'}^{\dot\beta}+\bar{\lambda}^{\dot\beta})+\bar{\varepsilon}^{\dot\alpha}\bar{\sigma}_{\dot\alpha\beta\rho}(\lambda'^\beta+\lambda)^\beta]\partial_\sigma L, \tag{90}$$

$$\delta l^\alpha=\sqrt{2}\varepsilon^\alpha F_L+i\sqrt{2}\bar{\varepsilon}^{\dot\alpha}\sigma^{\alpha\mu}_{\dot\alpha}D_\mu L+$$
$$+\theta^{\rho\sigma}\Big[\sqrt{2}\varepsilon^\alpha W_\rho\partial_\sigma F_L+\frac{1}{2}\Big(i\varepsilon^\beta\sigma_{\beta\dot\beta\rho}\overline{\lambda'}^{\dot\beta}+i\bar{\varepsilon}^{\dot\beta}\bar{\sigma}_{\dot\beta\beta\rho}\lambda'^\beta+i\varepsilon^\beta\sigma_{\beta\dot\beta\rho}\bar{\lambda}^{\dot\beta}+$$
$$+i\bar{\varepsilon}^{\dot\beta}\sigma_{\dot\beta\beta\rho}\lambda^\beta\Big)\partial_\sigma l^\alpha-\frac{\sqrt{2}}{4}\bar{\varepsilon}^{\dot\alpha}\sigma^\mu_{\dot\alpha\alpha}\Gamma_{\mu\rho}\partial_\sigma L-i\frac{\sqrt{2}}{2}\bar{\varepsilon}^{\dot\alpha}\sigma^\mu_{\dot\alpha\alpha}\Big(V'_\rho\partial_\sigma V'_\mu+ \tag{91}$$
$$+V_\rho\partial_\sigma V_\mu+V'_\rho F'_{\sigma\mu}+V_\rho F_{\sigma\mu}\Big)L\Big],$$

$$
\begin{aligned}
\delta F_L &= i\sqrt{2}\bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha}D_{\mu}l^{\alpha} - 2g\bar{\varepsilon}^{\dot{\alpha}}\bar{\lambda}_{\dot{\alpha}}L - g'\Upsilon\bar{\varepsilon}^{\dot{\alpha}}\bar{\lambda}'_{\dot{\alpha}}L + \\
&+\theta^{\rho\sigma}[\bar{\varepsilon}^{\dot{\alpha}}\bar{\lambda}_{\dot{\alpha}}W_{\rho}W_{\sigma}L - g\bar{\varepsilon}^{\dot{\alpha}}\partial_{\sigma}\bar{\lambda}_{\dot{\alpha}}V'_{\rho}L + g\bar{\varepsilon}^{\dot{\alpha}}\partial_{\rho}\bar{\lambda}_{\dot{\alpha}}\partial_{\sigma}L - \\
&-\frac{i}{4}g'\Upsilon\bar{\varepsilon}^{\dot{\alpha}}\bar{\lambda}'_{\dot{\alpha}}W_{\rho}W_{\sigma}L + \frac{1}{2}g'\Upsilon\bar{\varepsilon}^{\dot{\alpha}}\bar{\lambda}'_{\dot{\alpha}}W_{\rho}\partial_{\sigma}L + g'\Upsilon\bar{\varepsilon}^{\dot{\alpha}}\partial_{\sigma}\bar{\lambda}'_{\dot{\alpha}}V_{\rho}L - \\
&-\frac{i}{2}g\Upsilon\bar{\varepsilon}^{\dot{\alpha}}\partial_{\rho}\bar{\lambda}'_{\dot{\alpha}}\partial_{\sigma}L - i\frac{\sqrt{2}}{4}\bar{\varepsilon}^{\dot{\alpha}}\sigma^{\mu}_{\dot{\alpha}\alpha}\Gamma_{\mu\rho}\partial_{\sigma}l^{\alpha} - \\
&-\frac{\sqrt{2}}{2}\bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha}(V'_{\rho}\partial_{\sigma}V'_{\mu} + V_{\rho}\partial_{\sigma}V_{\mu} + V'_{\rho}F'_{\mu\sigma} + V_{\rho}F_{\mu\sigma})l^{\alpha} + \\
&+\frac{\sqrt{2}}{4}\bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha}W_{\rho}W_{\sigma}\partial_{\mu}l^{\alpha} - \bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha}W_{\rho}W_{\mu}\partial_{\sigma}l^{\alpha} - \frac{\sqrt{2}}{4}\bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha}W_{\mu}W_{\rho}W_{\sigma}l^{\alpha} + \\
&+(i\varepsilon^{\beta}\sigma_{\beta\dot{\beta}\rho}(\bar{\lambda}'^{\dot{\beta}} + \bar{\lambda}^{\dot{\beta}}) + i\bar{\varepsilon}^{\dot{\beta}}\bar{\sigma}_{\dot{\beta}\beta\rho}(\lambda'^{\beta} + \lambda^{\beta}))\partial_{\sigma}F_L],
\end{aligned}
\tag{92}
$$

$$
\delta R = \sqrt{2}\varepsilon_{\alpha}r^{\alpha} - \frac{1}{2}\theta^{\rho\sigma}\partial_{\rho}R(\varepsilon^{\alpha}\sigma^{\mu}_{\alpha\dot{\beta}}\bar{\lambda}^{\dot{\beta}} + \bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\dot{\alpha}\beta}\lambda^{\beta}),
\tag{93}
$$

$$
\begin{aligned}
\delta r^{\alpha} &= \sqrt{2}\varepsilon^{\alpha}F_R + i\sqrt{2}\bar{\varepsilon}^{\dot{\alpha}}\sigma^{\mu}_{\dot{\alpha}\alpha}D_{\mu}r + \\
&+\theta^{\rho\sigma}\left[\frac{i}{2}\sqrt{2}\bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha}D_{\rho}RF'_{\mu\sigma} + \frac{\sqrt{2}}{2}\varepsilon^{\alpha}\partial_{\rho}F_R V'_{\sigma} + \right. \\
&\left.+\frac{1}{2}\left(\bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}_{\dot{\alpha}\beta\rho}\lambda'^{\beta} + \varepsilon^{\alpha}\sigma_{\alpha\dot{\beta}\rho}\bar{\lambda}'^{\dot{\beta}}\right)\partial_{\sigma}r^{\alpha}\right],
\end{aligned}
\tag{94}
$$

$$
\begin{aligned}
\delta F_R &= i\sqrt{2}\bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha}D_{\mu}r^{\alpha} - g'\Upsilon\bar{\varepsilon}^{\dot{\alpha}}\bar{\lambda}'_{\dot{\alpha}}R + \\
&+\theta^{\rho\sigma}[-\frac{1}{2}g'\Upsilon\bar{\varepsilon}^{\dot{\alpha}}D_{\rho}\bar{\lambda}'_{\dot{\alpha}}\partial_{\sigma}R + (\bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}_{\dot{\alpha}\alpha\rho}\lambda'^{\alpha} + \varepsilon^{\alpha}\sigma_{\alpha\dot{\alpha}\rho}\bar{\lambda}'^{\dot{\alpha}})\partial_{\rho}F_R + \\
&+\frac{i}{2}\sqrt{2}\bar{\varepsilon}^{\dot{\alpha}}\bar{\sigma}^{\mu}_{\dot{\alpha}\alpha}D_{\rho}r^{\alpha}F'_{\mu\sigma} + \frac{\sqrt{2}}{2}\varepsilon^{\beta}\sigma^{\mu}_{\alpha\beta}\partial_{\sigma}V'_{\rho}V'_{\mu}r^{\alpha}].
\end{aligned}
\tag{95}
$$

Where the ε^{α} is the corresponding parameter of the above generalized supersymmetry transformations. It is worth to note that Higgs partners and the quarks fields transformations are similar to that of the leptons except for some additional group index and similary the Higgs fields transformations like lepton partners.

5. SUMMARY

Through this paper we have investigated the Lagrangian of the Minimal Supersymmetric Standard Model in the framework of the noncommutative space time. In particular, we have derived the noncommutative supersymmetric transformations of each component of the Lagrangian of the model (physical and their associate auxiliary fields) at the first order of the noncommutative parameter $\theta^{\mu\nu}$ and have built the corresponding noncommutative invariant Lagrangian.

6. APPENDIX

We use the following conventions for the generators of the $SU(2)$ gauge group T^a:

$$T^a = \frac{1}{2}\sigma^a$$

where σ^a is the Pauli matrices, $a = 1, 2, 3$. Hence, the generators of the $SU(2)$ gauge group obey the following relations:

$$[T^a, T^b] = i\varepsilon^{abc}T^c, \qquad tr(T^a T^b) = \frac{1}{2}\delta^{ab}.$$

Spinorial derivatives are defined as follows:

$$\frac{\partial}{\partial\theta^\alpha}\theta^\beta = \delta^\beta_\alpha, \qquad \frac{\partial}{\partial\bar{\theta}^{\dot\alpha}}\bar{\theta}^{\dot\beta} = \delta^{\dot\beta}_{\dot\alpha},$$

and the superspace integrations read

$$\int d^8 z = \int d^4 x \mathfrak{D}^2 \bar{\mathfrak{D}}^2, \qquad \int d^6 z = \int d^4 x \mathfrak{D}^2.$$

with

$$\mathfrak{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha}\partial_\mu, \qquad \bar{\mathfrak{D}}_{\dot\alpha} = \frac{\partial}{\partial\bar{\theta}^\alpha} + i\bar{\theta}^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu.$$

Furthermore, we use the following notations:

$$\sigma^{\mu\nu}_{\alpha\beta} = \frac{i}{2}(\sigma^\mu_{\alpha\dot\alpha}\bar{\sigma}^{\nu\dot\alpha}_\beta - \sigma^\nu_{\alpha\dot\alpha}\bar{\sigma}^{\mu\dot\alpha}_\beta), \qquad \bar{\sigma}^{\mu\nu}_{\dot\alpha\dot\beta} = \frac{i}{2}(\bar{\sigma}^{\mu\alpha}_{\dot\alpha}\sigma^\nu_{\alpha\beta} - \bar{\sigma}^{\nu\alpha}_{\dot\alpha}\bar{\sigma}^\mu_{\alpha\beta}).$$

The field-strength tensors and superfield-strength are defined by

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu]$$

$$F'_{\mu\nu} = \partial_\mu V'_\nu - \partial_\nu V'_\mu,$$

and

$$F^\alpha = 4ge^{-i\theta\sigma^\mu\bar{\theta}\partial_\mu}(-2\lambda^\alpha - 2\theta^\alpha D_V + \sigma^{\mu\nu\alpha\beta}\theta_\beta F_{\mu\nu} - 2i\sigma^{\mu\alpha}_{\dot\alpha}D_\mu\bar{\lambda}^{\dot\alpha}\theta^2)$$

$$F'^\alpha = 4g'e^{-i\theta\sigma^\mu\bar{\theta}\partial_\mu}\left(-2\lambda'^\alpha - 2\theta^\alpha D_{V'} + \sigma^{\mu\nu\alpha\beta}\theta_\beta F'_{\mu\nu} - 2i\sigma^{\mu\alpha}_{\dot\alpha}D'_\mu\bar{\lambda}'^{\dot\alpha}\theta^2\right).$$

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