Oscillations of Pseudo-Dirac Neutrinos and Supernovae

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Pseudo-Dirac Neutrinos & SuperNovæ

Oscillations of Neutrinos

- ***Solar Neutrino Problem*** $\Delta m^2_{sol} = \Delta m^2_{21} \simeq 7.6\times 10^{-5} eV^2$ & $\sin^2 2\theta_{12} \simeq 0.3$

Super-Kamiokande Collaboration, Phys. Rev. D83 (2011)

- ***Atmospheric Neutrino Problem*** $\Delta m^2_{atm} = \Delta m^2_{31} \simeq 2.45\times 10^{-3} eV^2$ & $\sin^2 2\theta_{23} \simeq 0.5$

MINOS Collaboration, Phys. Rev. Lett 106 (2011)

- ***Short Baseline Experiments*** $\Delta m^2_{SBL} = \Delta m^2_{41} \simeq 1 eV^2$

S. Schael et al., Phys. Rept. 427 (2006) & J. Kopp et al., JHEP (2013)

- ***Low Flux of High Energy Astrophysical Muon Neutrinos*** $\Delta m^2_{GRB} = \Delta m^2 \leq 10^{-12} eV^2$

IceCube Collaboration, Nature 484 (2012) & S. Pakvasa et al; Phys. Rev. Lett. 110 (2013)

- ***Contribute to Supernova Shock Revival***

H.Th. Janka, A&A 368 (2001)

Oscillations of Neutrinos in Supernovae

- $\nu_{\mu,\tau} \to \nu_e$ RSP theoretical study — C.S. Lim & W.L. Marciano, INS report 645, (1987)
- $\nu_{\mu,\tau} \to \nu_e$ RSP $9eV^2 \le \Delta m^2$ & $R \approx 1.6$ — E.Kh. Akhmedov et al, Phys. Rev. D 55 (1997)
- $\nu_{\mu,\tau} \to \nu_e$ MSW $10^3 eV^2 \le \Delta m^2 \le 10^5 eV^2$ — G.M. Fuller et al., Astro.J.389 (1992)
- **Sterile Neutrino Oscillations :** To include dark matter candidate

  $10eV \le \Delta m_{21}^2 \le 20keV$ & $10^{-9} \le \sin^2 2\theta_s \le 0.01$ — L.W. MacKenzie et al, arXiv:1405.6101

  $\Delta m^2 \sim 1keV$ & $\sin^2 2\theta_s \simeq 10^{-9}$ & $R = 2$ — J. Hidaka & G.M. Fuller, Phys.Rev.D76 (2007)
- $\Delta m^2 \sim 1eV^2$ **suppresses neutrino heating** — M.R. Wu et al, Phys.Rev.D89, (2014)
- **Collective Neutrino Oscillation** enhancing heating by not more than a few percent in the most optimistic case — B. Dasgupta et al, Phys. Rev. D 85 (2012)

Pseudo-Dirac Neutrinos:

- **Atmospheric Neutrino Problem :** but problems with SuperKam results.

M. Kobayashi & C.S. Lim, Phys.Rev. D64 (2001)

- **Solar Neutrino Oscillations:** must have $\Delta m_s^2 \leq 10^{-9} eV^2$

deGouvea et al. Phys.Rev.D80 (2009)

- **Neutrinoless double beta decay would be highly suppressed**

J.F. Beacom et al; Phys.Rev.Lett. 92 (2004)

- **UHE neutrinos events at IceCube coincident with gamma ray bursts (GRBs)**

$$10^{-12} eV^2 \leq \Delta m_s^2 \leq 10^{-18} eV^2$$

J.F. Beacom et al; Phys.Rev.Lett. 92 (2004)

A. Esmaili, Phys.Rev. D81 (2010) and A. Esmaili & Y. Farzan, JCAP 1212 (2012)

S. Pakvasa et al, Phys. Rev. Lett. 110 (2013)

A.S. Joshipura & S.D. Rindani; Phys.Rev. D89 (2014)

Oscillations of Neutrinos

Physical vs. Interaction States:

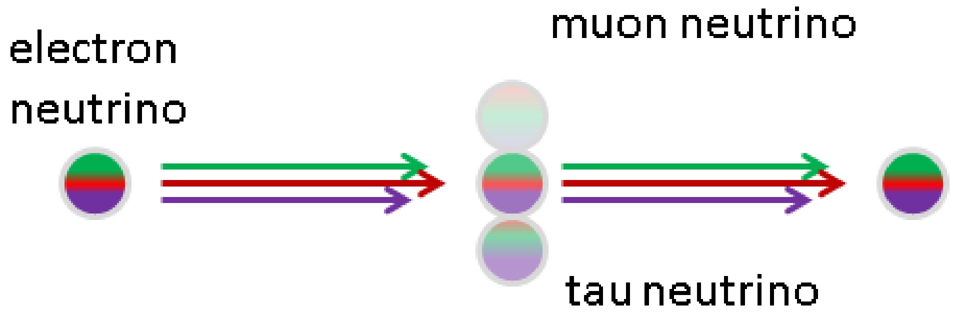
Neutrinos are produced as field states via weak interactions $\nu^{(f)} \equiv \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}$

But propagate as mass states: $i\frac{d}{dt}\nu^{(P)}(t) = H\nu^{(P)}(t) \qquad \nu^{(P)} \equiv \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$

$$\nu^{(P)} = O^{+}\nu^{(f)} \qquad\qquad i\frac{d}{dx}\nu^{(f)} = OHO^{+}\nu^{(f)}$$

Due to their differing masses, $\nu^{(P)}$ will travel at slightly different velocities. Then, are detected as other field states:

$$\nu^{(f)}_{new} \equiv \begin{pmatrix} \nu'_\alpha \\ \nu'_\beta \end{pmatrix}$$



Oscillations of Neutrinos

In Vacuum: no interactions (Bilenky S.M. arXiv:1208.2497)

For field states

$$H = \left(\left|P\right| + \frac{m_1^2 + m_2^2}{4\left|P\right|} \right) - \frac{\Delta}{4\left|P\right|}\sigma_3 \qquad \Delta = m_2^2 - m_1^2$$

For mass states

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad H_V = UHU^+ = \left|P\right| + \frac{m_1^2 + m_2^2}{4\left|P\right|} + \frac{\Delta}{\left|P\right|} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

So probability for transformation after distance x:

$$P(\nu_\alpha \to \nu_\beta, x) = \left| \left\langle \nu_\beta\left(x\right) \middle| \nu_\alpha\left(0\right) \right\rangle \right|^2 = \sin^2 2\theta \times \sin^2 \frac{\Delta \times x}{4E}$$

Not negligible if:

$$\sin^2 2\theta \approx 1 \qquad x \geq L_m / 2$$

Oscillations of Neutrinos

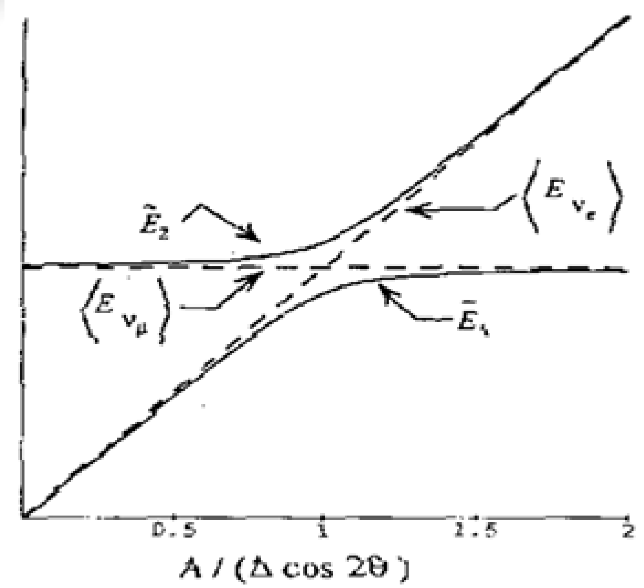
In Matter: Probability of transformations:

$$P(\nu_i \to \nu_j, x) = \sin^2 2\theta_m \times \sin^2 \frac{\pi . x}{L_m} : i \neq j$$

With matter angle:

$$\sin^2 2\theta_m = \frac{\Delta^2 \sin^2 2\theta}{(\Delta \cos 2\theta - A)^2 + \Delta^2 \sin^2 2\theta}$$

$$L_m = \frac{4\pi E / \Delta}{\sqrt{\cos^2 2\theta (1 - A / A_r)^2 + \sin^2 2\theta}} ; A_r = \Delta \cos 2\theta$$



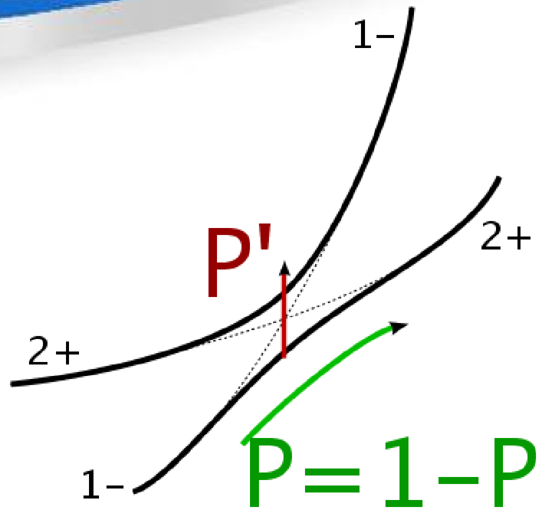
Mikheyev-Smirnov-Wolfenstein Effect:

$$A = \Delta \cos 2\theta = A_r \Rightarrow \overline{P}(\nu_e \to \nu_\mu) = \frac{1}{2} \sin^2 2\theta_m \approx \frac{1}{2}$$

Oscillations of Neutrinos

In Matter: *non-homogenous*

$$i\frac{d}{dx}\begin{pmatrix}\nu_m^{(1)}\\ \nu_m^{(2)}\end{pmatrix}=\begin{pmatrix}\frac{M_1^2}{2E} & i\frac{d\theta_m}{dx}\\ i\frac{d\theta_m}{dx} & \frac{M_2}{2E}\end{pmatrix}\begin{pmatrix}\nu_m^{(1)}\\ \nu_m^{(2)}\end{pmatrix}$$



Adiabatic case or $\frac{d\theta_m}{dx}$ *small:*

$$P_{ad}(\nu_e\to\nu_e)=1-P_{ad}(\nu_e\to\nu_\mu)=\frac{1}{2}\left[1+\cos 2\theta_m^{(C)}\cos 2\theta_m^{(D)}\right]$$

Non-Adiabatic case or $\frac{d\theta_m}{dx}$ *not small:*

$$P_{nad}(\nu_e\to\nu_\mu)=\frac{1}{2}\left[1-(1-2P_{LZ})\cos 2\theta_m^{(C)}\cos 2\theta_m^{(D)}\right]$$

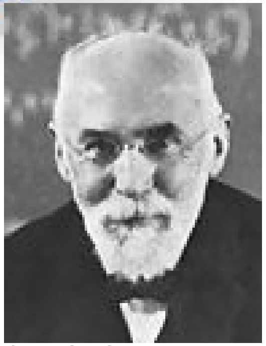
Landau-Stuckelberg-Zener Effect:

$$P_{LZ}=\exp\left(-\frac{\pi}{4}F\gamma_R\right)\qquad \gamma_R=\frac{\Delta}{E}\frac{\sin^2 2\theta}{\cos 2\theta}\left|\frac{d}{dx}\ln n_e\right|_R^{-1}$$

Oscillations of Neutrinos

Neutrino types: From left handed neutrinos to right handed ones, we have two possible ways:

$$\nu_- \xrightarrow{CPT} \overline{\nu}_+ \qquad \nu_- \xrightarrow{Lorentz} \nu_+$$



Hendrik A. Lorentz
1853-1928

Majorana neutrinos:

$$-L_m^{D+M} = \frac{1}{2}\overline{n}_L^C \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} n_L + h.c \qquad n_L \equiv \begin{pmatrix} \nu_{1,L} \\ \nu_{2,L} \\ \nu_{1,R}^C \\ \nu_{2,R}^C \end{pmatrix}$$

Ettore Majorana
1906-1938

Dirac neutrinos:

$$\begin{pmatrix} M_L = 0 & M_D \\ M_D^T & M_R = 0 \end{pmatrix}$$

Pseudo-Dirac neutrinos:

$$\begin{pmatrix} M_L \ll M_D & M_D \\ M_D^T & M_R \ll M_D \end{pmatrix}$$

Paul A. M. Dirac
1902-1984

Pseudo-Dirac Neutrinos

General Formalism: We write sterile neutrino as right handed one and we use the following basis.

$$i\frac{d}{dt}\nu(t) = H\nu(t) \qquad H = H(\nu) + H(int) + H(F) \qquad \nu = \left(\nu_L, \nu_R^C, \nu_L^C, \nu_R\right) = \left(\nu_e, \overline{\nu}_R, \overline{\nu}_e, \nu_R\right)$$

$$H = \begin{pmatrix} -\frac{1}{2}\Delta C2\theta + V_L & \frac{1}{2}\Delta S2\theta & 0 & \mu B \\ \frac{1}{2}\Delta S2\theta & \frac{1}{2}\Delta C2\theta - V_R & -\mu B & 0 \\ 0 & -\mu B & -\frac{1}{2}\Delta C2\theta - V_L & \frac{1}{2}\Delta S2\theta \\ \mu B & 0 & \frac{1}{2}\Delta S2\theta & \frac{1}{2}\Delta C2\theta + V_R \end{pmatrix}$$

$$S2\theta \equiv \sin 2\theta \qquad C2\theta \equiv \cos 2\theta \qquad \Delta \equiv \Delta m^2 / 2E \qquad V_L = \sqrt{2}\, G_F \left(n_e - \frac{1}{2} n_n\right); V_R = \sqrt{2} G \times n_e$$

Pseudo-Dirac Neutrinos

Possible Oscillations: *by equalizing the diagonal terms, we have 4 cases:*

$$\alpha = G/G_F \qquad n_n = \rho\left(1 - Y_e\right)N_A ; n_e = \rho Y_e N_A$$

$$\sqrt{2}\Delta\cos 2\theta = G_F N_A \rho F\left(Y_e\right) \qquad F\left(Y_e\right) = \begin{cases} \left(3+2\alpha\right)Y_e - 1 \leftrightarrow \overline{\nu}_R \longrightarrow \nu_e \left(MSW1\right) \\ -\left(3+2\alpha\right)Y_e + 1 \leftrightarrow \nu_R \longrightarrow \overline{\nu}_e \left(MSW2\right) \\ \left(3-2\alpha\right)Y_e - 1 \leftrightarrow \nu_R \longrightarrow \nu_e \left(RSP1\right) \\ -\left(3-2\alpha\right)Y_e + 1 \leftrightarrow \overline{\nu}_R \longrightarrow \overline{\nu}_e \left(RSP2\right) \end{cases}$$

MSW *for Mikheyev-Smirnov-Wolfenstein* ***RSP*** *for Resonant Spin Precession*

$$\frac{\Delta m^2}{2E}\cos 2\theta = \frac{1}{\sqrt{2}} G_F N_A \rho \left[\pm\left(3 \pm 2\frac{G}{G_F}\right)Y_e \mp 1\right]$$

Pseudo-Dirac Neutrinos

Possible Oscillations:

$$\frac{\Delta m^2}{2E}\cos 2\theta = \frac{1}{\sqrt{2}} G_F N_A \rho \left[\pm \left(3 \pm 2\frac{G}{G_F} \right) Y_e \mp 1 \right]$$

We take $\Delta m^2 \leq 10^{-9} eV^2$ *from solar neutrinos experiments (deGouvea et al. Phys.Rev.D80 2009) and* $E \approx 10 MeV$*; So the right hand side is close to zero:*

$$\rho \approx 10^7 \sim 10^{12} g / cm^3 \Rightarrow \left[\pm \left(3 \pm 2\frac{G}{G_F} \right) Y_e \mp 1 \right] \approx 0$$

For MSW: $\left(3+2\alpha\right) Y_e - 1 \approx 0 \,\&\, 0 \leq Y_e \leq 1/2 \Rightarrow \alpha \geq -1/2$

For RSP: $\left(3-2\alpha\right) Y_e - 1 \approx 0 \,\&\, 0 \leq Y_e \leq 1/2 \Rightarrow \alpha \leq 1/2$

Pseudo-Dirac Neutrinos

Possible Oscillations: *We have 3 different regions*

$$\begin{cases} \alpha \geq 1/2 \Rightarrow \bar{\nu}_R \longrightarrow \nu_e \left(MSW1\right) \& \ \nu_R \longrightarrow \bar{\nu}_e \left(MSW2\right) \\ 1/2 \geq \alpha \geq -1/2 \Rightarrow \left(MSW1\right) \& \left(MSW2\right) \& \left(RSP1\right) \& \left(RSP2\right) \\ -1/2 \geq \alpha \Rightarrow \nu_R \longrightarrow \nu_e \left(RSP1\right) \bar{\nu}_R \longrightarrow \bar{\nu}_e \left(RSP2\right) \end{cases}$$

And the transition probabilities are

$$P\left(v_\alpha \rightarrow v_\beta\right) = \frac{1}{2} - \frac{1}{2}\cos 2\theta_m^{(C)} \cos 2\theta_m^{(D)} \left(1 - 2P_{LZ}\right)$$

$$\cos 2\theta_m \left(RSP\right) = \frac{\Delta \cos 2\theta - \sqrt{2} G_F N}{\sqrt{\left(2\mu B\right)^2 + \left(\Delta \cos 2\theta - \sqrt{2} G_F N\right)^2}}$$

$$\cos 2\theta_m \left(MSW\right) = \frac{\Delta \cos 2\theta - \sqrt{2} G_F N}{\sqrt{\left(\Delta \sin 2\theta\right)^2 + \left(\Delta \cos 2\theta - \sqrt{2} G_F N\right)^2}}$$

Pseudo-Dirac Neutrinos

Possible Oscillations: $$P\left(v_\alpha \to v_\beta\right) = \frac{1}{2} - \frac{1}{2}\cos 2\theta_m^{(C)} \cos 2\theta_m^{(D)} \left(1 - 2P_{LZ}\right)$$

With $$P_{LZ}^{MSW} = \exp\left\{-\left(\pi / 2\Delta\right)\frac{\sin^2 2\theta}{\cos 2\theta}\left|\frac{d}{dr}\ln N\right|_{res}^{-1}\right\}$$

$$P_{LZ}^{RSP} = \exp\left\{-(\pi / 2)\left(2\mu B\right)^2 \frac{1}{\Delta}\frac{1}{\cos 2\theta}\left|\frac{d}{dr}\ln N\right|_{res}^{-1}\right\}$$

And: $$N = \begin{cases} N\left(MSW1\right) = \rho N_A\left\{\left(\frac{3}{2} + \alpha\right)Y_e - \frac{1}{2}\right\} \\ N\left(MSW2\right) = -\rho N_A\left\{\left(\frac{3}{2} + \alpha\right)Y_e - \frac{1}{2}\right\} \\ N(RSP1) = \rho N_A\left\{\left(\frac{3}{2} - \alpha\right)Y_e - \frac{1}{2}\right\} \\ N(RSP2) = -\rho N_A\left\{\left(\frac{3}{2} - \alpha\right)Y_e - \frac{1}{2}\right\} \end{cases}$$

And we have $$\left|\frac{d}{dr}\ln N\right| = \left|\frac{d}{dr}\ln\left(\rho N_A\left\{\pm\left(\frac{3}{2} \pm \alpha\right)Y_e \mp \frac{1}{2}\right\}\right)\right| \approx \left|\frac{d}{dr}\ln \rho\right|$$

Pseudo-Dirac Neutrinos

Possible Oscillations: $$P\left(\nu_\alpha \to \nu_\beta\right) = \frac{1}{2} - \frac{1}{2}\cos 2\theta_m^{(C)} \cos 2\theta_m^{(D)} \left(1 - 2P_{LZ}\right)$$

We start with the MSW type of oscillations $P_{LZ}(MSW) \approx 1$ *unless* $\left|\frac{d}{dr}\ln\rho\right|_{res} \leq 10^{-16}$

Production reactions are at neutrino-sphere (Y_e goes from 0 to ½):

$$C : \rho \approx 10^{12} g \times cm^{-3} \Rightarrow G_F N \approx -10^{-3} eV$$

And detection at the Wilson bubble:

$$D : \rho \approx 10^{6} \sim 10^{7} g \times cm^{-3} \Rightarrow G_F N \approx 10^{-9} \sim 10^{-8} eV$$

This gives:

$$\cos 2\theta_m^{(C)} \approx 1 \,\&\, \cos 2\theta_m^{(D)} \approx -1 \Rightarrow P\left(MSW1\right) \approx 0$$

And also:

$$N\left(MSW2\right) = -N\left(MSW1\right) \Rightarrow P\left(MSW2\right) \approx 0$$

Pseudo-Dirac Neutrinos

Possible Oscillations: $P\left(v_{\alpha} \to v_{\beta}\right) = \frac{1}{2} - \frac{1}{2}\cos 2\theta_m^{(C)} \cos 2\theta_m^{(D)} \left(1 - 2P_{LZ}\right)$

We consider now the RSP type of oscillations

$$\mu_v \approx 10^{-12}\mu_B; B_{res} \approx 10^{12}G; \Delta m^2 \approx 1eV^2$$

$$P_{LZ}^{RSP} \approx \exp\left\{-0.2 \cdot 10^{-11}\left(\frac{eV^2}{10^{-9}\Delta m^2}\right)\left(\frac{E_\nu}{10MeV}\right)\left(\frac{\mu_\nu B_{res}}{\mu_B \times G}\right)^2\left(10^{-5}cm\left|\frac{d}{dr}\ln\rho\right|_{res}^{-1}\right)\right\}$$

We take the profile:

$$B(r) \approx \left(r_0 / r\right)^n \times 10^{12}G; r_0 = 10km; r = 400km; \mu_\nu \approx 10^{-12}\mu_B \Rightarrow \mu B \approx 10^{-12}eV$$

And as for the MSW case, we consider creation point C and detection one D, where we have the $G_F N$ term dominant in $\cos 2\theta$ expressions, and so we get: $P_{LZ}^{RSP} \approx 1$

and $P\left(RSP1\right) = P\left(RSP2\right) \approx 1$ unless $\mu_\nu B_{res} \geq 10^6 \mu_B G$

Pseudo-Dirac Neutrinos

Possible Oscillations: *We summarize:*

$$
\begin{cases}
\alpha \le -1/2 \Rightarrow \begin{cases} P\left(\nu_R \to \nu_e\right) = P\left(RSP1\right) \approx 1 \\ P\left(\overline{\nu}_R \to \overline{\nu}_e\right) = P\left(RSP2\right) \approx 1 \end{cases} \\
-1/2 \le \alpha \le 0 \Rightarrow \begin{cases} P\left(\nu_R \to \nu_e\right) = P\left(RSP1\right) \approx 1 \\ P\left(\overline{\nu}_R \to \overline{\nu}_e\right) = P\left(RSP2\right) \approx 1 \\ P\left(\nu_R \to \overline{\nu}_e\right) = \left[1 - P\left(RSP1\right)\right] P\left(MSW2\right) \approx 0 \\ P\left(\overline{\nu}_R \to \nu_e\right) = \left[1 - P\left(RSP2\right)\right] P\left(MSW1\right) \approx 0 \end{cases} \\
0 \le \alpha \le 1/2 \Rightarrow \begin{cases} P\left(\nu_R \to \nu_e\right) = \left[1 - P\left(MSW2\right)\right] P\left(RSP1\right) \approx 1 \\ P\left(\overline{\nu}_R \to \overline{\nu}_e\right) = \left[1 - P\left(MSW1\right)\right] P\left(RSP2\right) \approx 1 \\ P\left(\nu_R \to \overline{\nu}_e\right) = P\left(MSW2\right) \approx 0 \\ P\left(\overline{\nu}_R \to \nu_e\right) = P\left(MSW1\right) \approx 0 \end{cases} \\
1/2 \le \alpha \Rightarrow \text{all probabilities are zero}
\end{cases}
$$

Conclusion:

- Only RSP oscillations are relevant in our case
- If we use the estimation of Akhmedov: E.Kh. Akhmedov, Phys.Rev.D55 (1997)

$$R_{RSP} \approx \frac{Y_n + Y_p \left(T_{\nu_s} / T_{\nu_s}\right)^2}{Y_n + Y_p \left(T_{\bar{\nu}_e} / T_{\nu_s}\right)^2} \approx 40$$

To compare to a factor 2 when using $\nu_{\mu,\tau}$. G.M. Fuller et al., Astro.J.389 (1992)

- For the cross section, we have

Following the estimation made by: J. Hidaka & G.M. Fuller, Phys.Rev.D76 (2007)

- Oscillations between sterile and active neutrinos can fulfill the conditions for shock revival by neutrino heating in core-collapse supernovae. H.Th. Janka, A&A 368 (2001)
- Need computations instead of estimations

First and foremost, because we exist, we need an explosion:

so:

"Put your trust in God and keep your powder dry!"

Oliver Cromwell , 1649 or William Blacker, 1834