

وزارة التعليم العالي والبحث العلمي

Ministry of Higher Education and Scientific Research



Mohamed Khider Biskra University  
Faculty of Sciences and Technology  
Electrical Engineering Department

Field: Electrotechnique

Option: Electrical Grids

Ref: .....

A Dissertation for the Fulfillment of the  
Requirement of a Master's Degree

*Theme*

***Power Grid Optimization Using PSO and  
Genetic Algorithms***

Presented by:

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Presented on: 29 May 2016

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*Academic Year: 2015 / 2016.*

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**Favorable opinion of the supervisor:**

Dr. Djemai NAIMI

**Favorable opinion of the Jury President**

Pr. Souri-Mohamed MIMOUNE

**Stamp and signature**

**ملخص**

في هذه المذكرة، نقدم المنهجية المستعملة في تطوير برنامج التدفق الأمثل للقدرة الكهربائية في الشبكة الكهربائية باستعمال طريقتين : الخوارزميات الجينية و أمثلة سرب الجسيمات عن طريق الماتلاب لمجموعة من الدوال الهادفة المتعلقة بالشبكة الكهربائية، وهذه الدوال لا تقتصر فقط على تكلفة الإنتاج في المحطات الحرارية، فهناك عديد الجوانب منها جانب الضياعات في خطوط نقل الطاقة وكذلك الجانب البيئي الذي أصبح من أهم الانشغالات بسبب الغازات المنبعثة والتي تمثل خطرا كبيرا على البيئة. ستنم فقط أمثلة دالتين هادفتين من الثلاثة المذكورة سابقا باستعمال طريقتي الخوارزميات الجينية وأمثلة سرب الجسيمات، هما تكلفة الإنتاج وانبعاث الغازات. نتائج المحاكاة المطبقة على الشبكة الكهربائية ذي الثلاثين قضيبا للتجميع كانت مرضية وتحترم جميع العوائق المفروضة سابقا.

**الكلمات المفتاحية:** التدفق الأمثل للقدرة الكهربائية، الخوارزميات الجينية، أمثلة سرب الجسيمات، ماتلاب.

**Abstract**

In this paper, we present the methodology used in the development of the Optimal Power Flow (OPF) program in the grid using two metaheuristic methods : Genetic Algorithms (GAs) and Particle Swarm Optimization (PSO) developed in MATLAB for objective functions relating to the grid, and these functions are not limited to the energy production cost in power plants, there are many aspects, including losses in electricity transmission lines, as well as the environmental aspect that has become one of the biggest concerns because the gases emitted by power plants represent a significant danger to the environment. Two only functions among the three mentioned ones are going to be optimized using the GA and PSO methods, which are: the production cost and the gases emission functions. The simulation results applied on the 30 Bus grid are very satisfactory and respecting all the constraints imposed previously.

**Keywords:** Optimal Power Flow, Genetic Algorithms, Particle Swarm Optimization, MATLAB.

## **Résumé**

Dans ce mémoire, nous présentons la méthodologie utilisée dans le développement de l'écoulement de puissance optimale dans le réseau électrique en utilisant deux méthodes métaheuristiques: Les Algorithmes Génétiques (AG), Optimisation par Essaim Particulaire (OEP) développées sous MATLAB pour des fonctions objectives relatives au réseau électrique, et ces fonctions ne se limitent pas au coût de la production d'énergie dans les centrales thermiques, il y a de nombreux aspects, y compris les pertes dans les lignes de transmission d'électricité, ainsi que l'aspect environnemental qui a devenu l'un des préoccupations les plus importantes à cause des gaz émis par les centrales thermiques représentant un danger important pour l'environnement. Seulement, deux fonctions entre les trois celles mentionnées vont être optimisé en utilisant les méthodes AG et OEP, qui sont: la fonction du coût et la fonction d'émission. Les résultats de simulation appliqués sur le réseau de 30 JB sont très satisfaisants et respectant toutes les contraintes imposées préalablement.

**Mots-clés:** Ecoulement de puissance optimale, Algorithmes Génétiques, Optimisation par Essaim Particulaire.

## **Acknowledgments**

Firstly, I would like to thank and praise God above all, *Allah* the Almighty, for granting me light, patience and stamina to accomplish my research.

Secondly, I would like to express my sincere gratitude and deep appreciation to my supervisor *Dr. Djemai NAIMI* for his endless support, continuous encouragement, and excellent motivation. His guidance helped me in all the time throughout my dissertation since the beginning until the end. I would like to extend my deepest gratitude to the jury members starting from *Pr. Souri-Mohamed MIMOUNE* who I consider him as my father because he has been always there for me, also *Mrs. Fatiha KHELILI* for her kind heart and her patience with us. Not forgetting *Dr. Ahmed SALHI* for his help during my research.

I would like to express my deep appreciation for membership of *LGEB & LMSE* for their financial support, without which this research would not be possible.

Yet, I am immensely grateful to all my teachers in the Department of Electrical Engineering, at *Mohamed Khider University of Biskra*. I am equally grateful and profoundly indebted to those who have taught and trained me throughout the entire educational career of mine. I also extend my heartfelt thanks to my family for backing me up spiritually in the midst of writing this thesis and in my life in general.

Finally yet importantly, I owe a great many thanks to a great many people and well-wishers. This work would have truly been nothing but a distant reality without their assistance, concern, and advice.

## List of Figures

### *Chapter 1 : Optimization of Power Flow*

- Figure 1.1** Illustration of the local minimizer  $X_L^*$  and the global minimizer  $X^*$  ..... 07
- Figure 1.2:** Flow diagram of basic load flow algorithm..... 10

### *Chapter 2 : Optimization Methods*

- Figure 2.1:** Structure of a single population evolutionary algorithm..... 20
- Figure 2.2:** Flow chart of a simple Genetic Algorithm..... 23
- Figure 2.3:** Flow chart Of PSO..... 27

### *Chapter 3 : Comparative Analysis*

- Figure 3.1:** Diagram of Combustion..... 32
- Figure 3.2:** Bloc diagram represents the required conditions to form an optimization problem..... 34
- Figure 3.3:** Single-line diagram of the IEEE 30 Bus power grid..... 35
- Figure 3.4:** Optimal repartition of generated powers..... 36
- Figure 3.5:** Evolution of the objective function over generations..... 37
- Figure 3.6:** Verification of transited powers constraint..... 37
- Figure 3.7:** Verification of generated powers constraint..... 38
- Figure 3.8:** Verification of tensions constraint..... 38
- Figure 3.9:** Optimal repartition of generated powers..... 40
- Figure 3.10:** Evolution of the objective function over generations..... 40
- Figure 3.11:** Current best individual..... 41
- Figure 3.12:** Verification of transited powers constraint..... 41
- Figure 3.13:** Verification of generated powers constraint..... 41
- Figure 3.14:** Verification of tensions constraint..... 42
- Figure 3.15:** Optimal repartition of generated powers..... 44
- Figure 3.16:** Evolution of the objective function over iterations..... 44
- Figure 3.17:** Verification of tensions constraint..... 45
- Figure 3.18:** Optimal repartition of generated powers..... 46
- Figure 3.19:** Evolution of the objective function over iterations..... 47
- Figure 3.20:** Verification of tensions constraint..... 47

## List of Tables

<b>Chapter 1 : Optimization of Power Flow</b>	
<b>Table 1.1:</b>	Load flow problem variables..... 09
<b>Chapter 2 : Optimization Methods</b>	
<b>Chapter 3 : Comparative Analysis</b>	
<b>Table 3.1:</b>	Optimized cost function and non-optimized emission function values.. 36
<b>Table 3.2:</b>	Optimal repartition of generated powers..... 36
<b>Table 3.3:</b>	Non-optimized cost function and optimized emission function values.. 39
<b>Table 3.4:</b>	Optimal repartition of generated powers..... 40
<b>Table 3.5:</b>	Optimized cost function and non-optimized losses function values..... 43
<b>Table 3.6:</b>	Optimal repartition of generated powers..... 43
<b>Table 3.7:</b>	Optimized emission function and non-optimized losses function value 46
<b>Table 3.8:</b>	Optimal repartition of generated powers..... 46

## **List of Symbols & Acronyms**

<b>N</b>	Total number of buses
<b>N<sub>g</sub></b>	Number of generator including the generator at slack bus
<b>k</b>	Node
<b>V<sub>k</sub></b>	Voltage magnitude
<b>δ<sub>k</sub></b>	Voltage angle
<b>P</b>	Injected real power
<b>Q</b>	Injected reactive power
<b>GS</b>	Gauss-Seidel
<b>NR</b>	Newton-Raphson
<b>a<sub>i</sub>, b<sub>i</sub>, c<sub>i</sub></b>	Coefficients of the generated power for generator i
<b>CO<sub>2</sub></b>	Carbon dioxide
<b>NO, NO<sub>2</sub></b>	Nitrogen oxides
<b>SO<sub>2</sub></b>	Sulphur dioxide
<b>n<sub>g</sub></b>	Number of generators
<b>α<sub>i</sub>, β<sub>i</sub>, γ<sub>i</sub>, λ<sub>i</sub></b>	Coefficients of the generated power for the generator i
<b>P<sub>Gi</sub></b>	Generated active power at Bus I
<b>P<sub>load</sub></b>	Active load power
<b>Q<sub>load</sub></b>	Reactive load power
<b>P<sub>losses</sub></b>	Active transmission losses
<b>Q<sub>losses</sub></b>	Reactive transmission losses
<b>V</b>	Vector of bus voltage magnitude with lower and upper limits, V <sup>m</sup> and V <sup>M</sup>
<b>δ</b>	Vector of bus voltage phase angle with lower and upper limits, δ <sup>m</sup> and δ <sup>M</sup>
<b>P<sub>di</sub></b>	Demanded power at Bus i
<b>μ, λ</b>	Vectors of the Lagrange multipliers, respectively
<b>EAs</b>	Evolutionary Algorithms
<b>SI</b>	Swarm Intelligence
<b>SA</b>	Simulated Annealing
<b>V<sub>imax</sub></b>	Maximal tension at Bus i

<b><math>P_{Gmin}</math></b>	Minimal generated active power in the grid
<b><math>V_{imin}</math></b>	Minimal tension at Bus i
<b><math>P_{Gmax}</math></b>	Maximal generated active power in the grid
<b><math>Q_{Gmax}</math></b>	Maximal generated reactive power in the grid
<b><math>Q_{Gmin}</math></b>	Minimal generated reactive power in the grid
<b>X</b>	Design vector
<b>MATLAB</b>	Matrix Laboratory
<b>OPF</b>	Optimal Power Flow
<b>PF</b>	Power Flow
<b>GA</b>	Genetic Algorithm
<b>PSO</b>	Particle Swarm Optimization
<b>Fobj(X)</b>	Objective Function
<b><math>S_{Tr ij}</math></b>	Transited power
<b><math>S_{Tr ij max}</math></b>	Maximal transited power
<b><math>S_{ij}</math></b>	Apparent power
<b><math>S_{ij max}</math></b>	Maximal transmitted power
<b><math>F_C</math></b>	Optimized function of cost
<b><math>F_E</math></b>	Optimized function of emission

## Table of Contents

Aknowledgments	
List of Figures & Tables .....	I
List of Symbols & Acronyms.....	I II
Table of Contents.....	V
Preface.....	1
<b><i>Chapter 1: Optimization of Power Flow</i></b>	
Introduction .....	4
<b>1- Optimization problem.....</b>	<b>4</b>
<b>1.1 Definition .....</b>	<b>4</b>
<b>1.2 Elements of an optimization problem.....</b>	<b>4</b>
<b>1.2.1 Objective function .....</b>	<b>4</b>
<b>1.2.2 Design vector.....</b>	<b>4</b>
<b>1.2.3 Design constraints.....</b>	<b>5</b>
<b>1.3 Formulation of an optimization problem.....</b>	<b>5</b>
<b>1.3.1 Dynamic optimization.....</b>	<b>5</b>
<b>1.3.1.1 Global optimization.....</b>	<b>6</b>
<b>1.3.1.2 Local optimization.....</b>	<b>6</b>
<b>1.4 Engineering applications of optimization.....</b>	<b>7</b>
<b>2. Optimization of power flow .....</b>	<b>7</b>
<b>2.1 Power flow (PF).....</b>	<b>8</b>
<b>2.1.1 Bus types and PF problem.....</b>	<b>8</b>
<b>2.1.2 PF solution.....</b>	<b>9</b>
<b>2.2 Problem formulation of OPF.....</b>	<b>11</b>
<b>2.2.1 Background and definitions.....</b>	<b>11</b>
<b>2.2.2 OPF objectives.....</b>	<b>12</b>
<b>2.2.3 Objective function.....</b>	<b>12</b>
<b>2.2.4 Equality constraints.....</b>	<b>13</b>
<b>2.2.5 Inequality constraints.....</b>	<b>14</b>
<b>2.2.6 Control variables.....</b>	<b>14</b>
Conclusion.....	15

<b>Chapter 2: Optimization Methods</b>	
Introduction .....	17
1. Optimization methods classification.....	17
1.1 Traditional algorithms .....	17
1.1.1 Newton based algorithm.....	18
1.1.2 Problems with traditional approaches.....	19
1.2 Evolutionary algorithms (EAs).....	19
2. Genetic algorithms (GAs).....	21
2.1 Overview of the GAs.....	21
2.2 GAs operators.....	21
2.3 GAs parameters.....	22
3. Particle swarm optimization (PSO).....	24
3.1 Overview of the PSO.....	24
3.2 Basic elements of the PSO.....	25
3.3 PSO algorithm.....	26
3.4 Advantages of the PSO.....	28
Conclusion .....	28

<b>Chapter 3: Comparative Analysis</b>	
Introduction.....	30
1. Choice of the objective functions to be optimized.....	30
1.1 Cost function.....	30
1.1.1 Economic dispatching.....	30
1.1.2 Dispatching problem formulation.....	31
1.2 Emission function.....	32
1.2.1 Environmental dispatching.....	32
1.2.2 Optimization problem formulation of environmental dispatching.....	33
1.3 Required conditions forming an optimization problem.....	34
2. Application.....	35
2.1 GA Method.....	35
2.1.1 Choice of parameters.....	35
2.1.2 Cost function $F_C$ .....	36
2.1.2.1 Verification of constraints.....	37
2.1.2.2 Discussion and Interpretation.....	38

<b>2.1.3</b> Emission function $F_E$ .....	39
<b>2.1.3.1</b> Verification of constraints.....	41
<b>2.1.3.2</b> Discussion and Interpretation.....	42
<b>2.2</b> PSO Method.....	43
<b>2.2.1</b> Choice of parameters.....	43
<b>2.2.2</b> Cost function.....	43
<b>2.2.2.1</b> Verification of constraints.....	45
<b>2.2.2.2</b> Discussion and Interpretation.....	45
<b>2.2.3</b> Emission function .....	46
<b>2.2.3.1</b> Verification of constraints.....	47
<b>2.2.3.2</b> Discussion and Interpretation.....	48
Conclusion .....	48
General Conclusion and Perspectives .....	50
References .....	52
Appendix.....	A

# **Preface**

## **Preface**

Nature has always been a source of inspiration. In recent years, new concepts, techniques and computational applications stimulated by nature are being continually proposed and exploited to solve a wide range of optimization problems in diverse fields. Various kinds of nature-inspired algorithms have been designed and applied, and many of them are producing high quality solutions to a variety of real-world optimization tasks. The success of these algorithms has led to competitive advantages and cost savings not only to the scientific community but also the society at large.

The use of nature-inspired algorithms stands out to be promising due to the fact that many real-world problems have become increasingly complex. The size and complexity of the optimization problems nowadays require the development of methods and solutions whose efficiency is measured by their ability to find acceptable results within a reasonable amount of time. Despite there is no guarantee of finding the optimal solution, approaches based on the influence of biology and life sciences such as evolutionary algorithms, neural networks, swarm intelligence algorithms, artificial immune systems, and many others have been shown to be highly practical and have provided state-of-the-art solutions to various optimization problems [RAY 09].

The purpose of this thesis is to present the techniques and applications of engineering optimization in a comprehensive manner. Some of the recently developed methods of optimization, such as genetic algorithms, particle swarm optimization are also discussed to make a comparison between the two methods to show their efficiency on the system.

With our ability to use an optimization and modeling code, such as MATLAB, we will be able to readily apply what we learn to real-life problems. MATLAB is very popular software that is useful in all areas of engineering. It is also used in a growing number of non-engineering fields. In this thesis, MATLAB program will be used as a computing solver for our optimization problem using two specific methods which are GA and PSO.

This present work consists of three chapters. Chapter 1 provides an introduction to the optimization problem and some of its applications in the engineering field. On the other hand, this chapter also treats the use of the optimization on the power flow by starting to understand the conventional load flow problem first, finishing by giving a main conclusion to this chapter.

In the second chapter, we will try to talk briefly about some methods used for the optimization, here we mean, conventional (traditional) methods and non conventional (evolutionary) methods, among those; we intend to speak intensely about the GA (Genetic Algorithm) and the PSO (Particle Swarm Optimization) techniques.

In the last chapter, a comparative study between two evolutionary methods which are GAs and PSO is concerned by applying them on a 30 Bus grid to see their efficiency on the system. But before that, we will try to deal with two main functions which are cost function (economic dispatching) and also emission function (environmental dispatching).

Finally, a general conclusion will briefly provide the main work of this thesis by giving also what we could summarize from our work with some perspectives for the future.

# **Chapter 1**

## **Optimization of Power Flow**

## **Introduction**

The objective of this chapter is to briefly introduce the optimization problem and some of its applications in the engineering field. On the other hand, this chapter also treats the use of the optimization on the power flow by starting to understand the conventional load flow problem first, finishing by giving a main conclusion to this chapter.

### **1- Optimization problem**

#### **1-1 Definition**

Optimization is the act of obtaining the best result under given circumstances. In design, construction, and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit [SIN 09].

#### **1-2 Elements of an optimization problem**

##### **1-2.1 Objective function**

The classical design procedure aims at finding an acceptable design, i.e. a design which satisfies the constraints. In general there are several acceptable designs, and the purpose of the optimization is to single out the best possible design. Thus, a criterion has to be selected for comparing different designs. This criterion, when expressed as a function of the design variables, is known as objective function.

The objective function is in general specified by physical or economical considerations. However, the selection of an objective function is not trivial, because what is the optimal design with respect to a certain criterion may be unacceptable with respect to another criterion.

##### **1-2.2 Design vector**

Any system is described by a set of quantities, some of which are viewed as variables during the design process, and some of which are *preassigned parameters* or are imposed by the environment. All the quantities that can be treated as variables are called *design or decision variables*, and are collected in the design vector **X**.

### 1-2.3 Design constraints

In practice, the design variables cannot be selected arbitrarily, but have to satisfy certain requirements. These restrictions are called design constraints. Design constraints may represent limitation on the performance or behavior of the system or physical limitations [AST 06].

### 1-3 Formulation of an optimization problem

A general definition of any optimization problem can be expressed as follows:

$$\left\{ \begin{array}{ll} \text{Minimize,} & f(u, x) & (1.1) \\ \text{Subject to:} & g(u, x) = 0 & (1.2) \\ & h(u, x) \leq 0 & (1.3) \end{array} \right.$$

where,

$f$  : objective function

$g$  : equality constraints

$h$  : inequality constraints

$u$  and  $x$  represent, respectively, a set of controllable and dependent variables.

The task is to minimize the objective function, while considering the constraints. The applicability of this definition to different classes of problems is by selecting suitable objective functions to be minimized, suitable sets of controllable quantities and suitable sets of equality and inequality constraints [TIN 87].

#### 1-3.1 Dynamic Optimization

Many optimization problems have objective functions that change over time and such changes in objective function cause changes in the position of optima. These types of problems are said to be *dynamic optimization problems*.

There are two techniques to solve optimization problems: Global and Local optimization techniques.

### 1-3.1.1 Global Optimization

A global minimizer is defined as  $x^*$  such that

$$f(x^*) \leq f(x), \forall x \in S \quad (1.4)$$

where,  $S$  is the search space and  $S = \mathbb{R}^n$  for unconstrained problems.

Here, the term global minimum refers to the value  $f(x^*)$ , and  $x^*$  is called the global minimizer. Some global optimization methods require a starting point  $z_0 \in S$  and it will be able to find the global minimizer  $x^*$  if  $z_0 \in S$ .

### 1-3.1.2 Local Optimization

A local minimizer  $x_L^*$  of the region  $L$ , is defined as

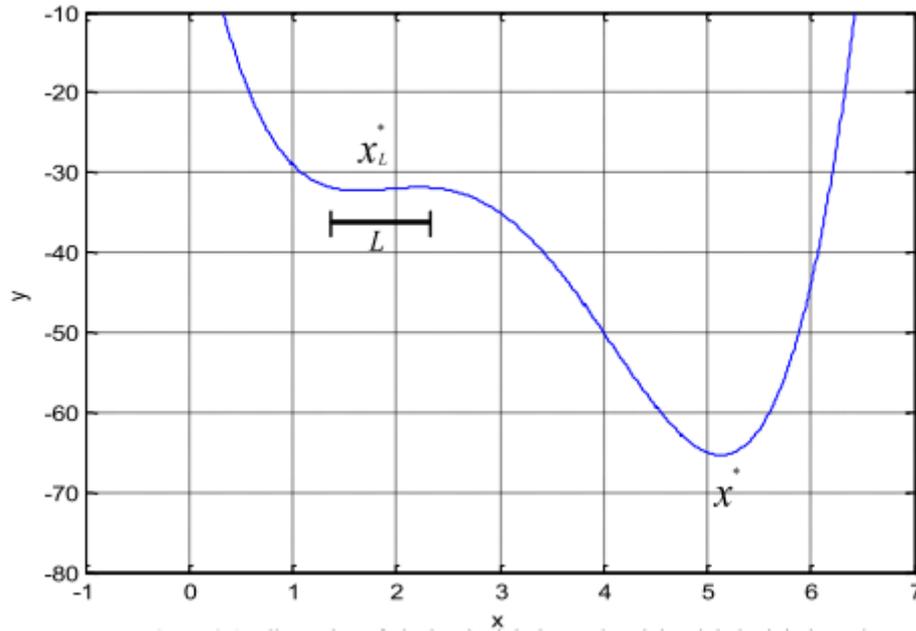
$$f(x_L^*) \leq f(x), \forall x \in L \quad (1.5)$$

where,  $L \in \mathbb{R}^n$ .

Here, a local optimization method should guarantee that a local minimizer of the set  $L$  is found.

Finally, local optimization techniques try to find a local minimum and its corresponding local minimizer, whereas global optimization techniques seek to find a global minimum or lowest function value and its corresponding global minimizer.

**Example:** Consider a function  $y = f(x) = x^4 - 12x^3 + 47x^2 - 75x + 10$ , and then the following (Figure 1.1) illustrates the difference between the global minimizer  $x^*$  and the local minimizer  $x_L^*$  [SAT 06].



**Figure 1.1:** Illustration of the local minimizer  $x_L^*$  and the global minimizer  $x^*$ .

#### 1-4 Engineering Applications Of Optimization [SIN 09]

Optimization, in its broadest sense, can be applied to solve any engineering problem. Some typical applications from different engineering disciplines indicate the wide scope of the subject:

1. Design of aircraft and aerospace structures for minimum weight
2. Finding the optimal trajectories of space vehicles
3. Optimum design of electrical networks
4. Optimal production planning, controlling, and scheduling
5. Planning of maintenance and replacement of equipment to reduce operating costs.

#### 2- Optimization of Power Flow

Optimal power flow is a program for scheduling power generation to minimize the operating cost. It optimizes the power flow solution of large-scale power system by setting the outputs of generators to minimize the total operating cost, while at the same time ensuring that load demands are satisfied and no transmission system elements are overloaded.

Optimal power flow studies are performed to achieve the following benefits:

- Cost saving due to reduced system losses
- Improved voltage control
- Improved system security; greater reserve margins

In summary, optimal power flow studies ensure an economical steady-state operation of the system while considering not only normal operating limits, but also violations that would occur during contingencies.

In order to understand optimal power flow, it is necessary to understand the conventional load flow problem first.

## **2-1 Power Flow (PF)**

A power flow is a study of steady-state operation of power system. Power flow is very important for network modeling, analysis and operation. Broadly speaking, it is a starting point for many static and dynamic analysis of power system. In addition, most notations and terms used in optimal power flow are introduced in power flow problem. Once the power flow is solved, further analysis such as optimal power flow can be easily performed [SHI 06].

### **2-1.1 Bus types and PF problem**

The power flow is the basic tool for investigating and computing these constraints. Power flow basically computes the following four variables: voltage magnitude ( $V_k$ ) and angle ( $\delta_k$ ) at each bus  $k$  (node), the real ( $P$ ) and reactive ( $Q$ ) power flows in each link of the network.

After the power flow has been solved, a power engineer can be able to comment on the bus voltage profile, the line flows and losses and last, but equally important, on the operating conditions of the system. These are the decisive and essential for operational planning, continuous evaluation of the operation of power systems and power system expansion, among others.

System buses are classified as voltage-controlled bus (or generator bus or PV bus), Load bus (PQ bus) or Slack (or swing) bus. At each bus, two variables are specified and the other two are unknown as shown in (Table 1.1) [NAI 15].

**Table 1.1:** Load flow problem variables.

<i>Bus type</i>	<i>Knowns</i>	<i>Unknowns</i>	<i>No. of Unknowns</i>
<b>Swing bus</b>	V, $\delta$	P, Q	2
<b>Voltage-controlled</b>	P, V	Q, $\delta$	$2(N_g - 1)$
<b>Load bus</b>	P, Q	V, $\delta$	$2(N - N_g)$
<b>Totals</b>	2N	2N	

**N:** total number of buses.

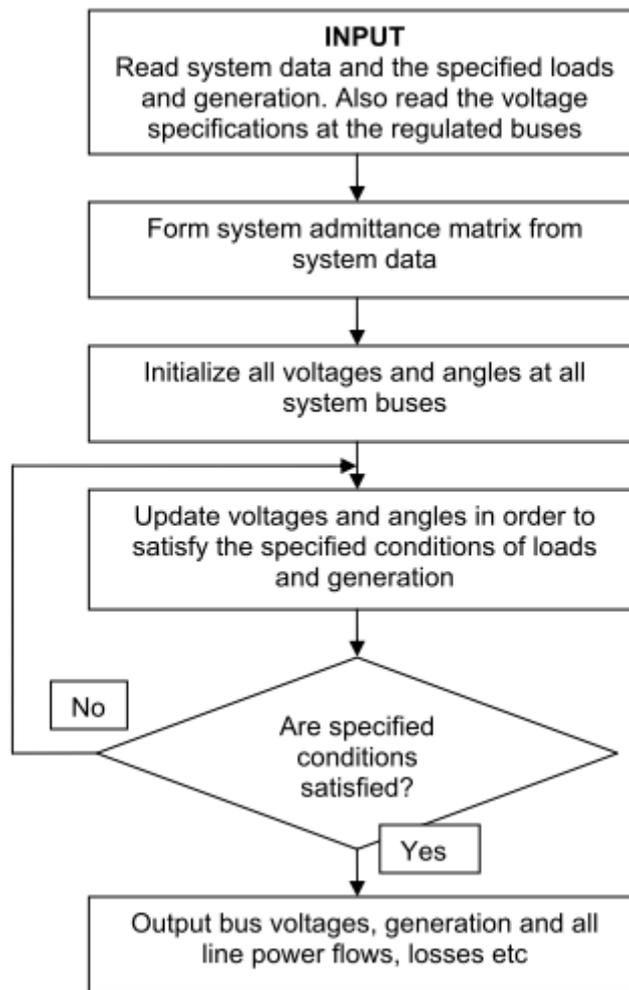
**N<sub>g</sub>:** number of generator including the generator at slack bus.

**V<sub>k</sub>:** voltage magnitude,  **$\delta_k$ :** voltage angle.

**P:** injected real power, **Q:** injected reactive power.

### 2-1.2 PF solution

Several algorithms are being used to solve the load-flow problem for large systems. A basic load flow algorithm runs according to the flowchart in (Figure 2).



**Figure 1.2:** Flow diagram of basic load flow algorithm.

The most common algorithms used for load flow solution are Gauss-Seidel (GS), Newton-Raphson (NR), Fast Decoupled and Quasi-Newton methods. Each of these methods has advantages and disadvantages.

GS method is the oldest of load flow solution methods. It is simple, reliable and can tolerate poor voltage and reactive power conditions. However, GS method requires a large number of iteration to converge which increases with the system size.

NR converges in several cases where GS method is failing to converge especially when the system size is increased. Furthermore, the number of iterations required for convergence with NR method is independent of the size of the system. The disadvantage of NR is that more computation time and computer storage is

required. The computation time increases with system size. NR also has convergence problems when the initial voltages are significantly different from their true values.

The fast decoupled load flow is a simplification of the NR method. It requires less iterations than NR method, and requires considerably less computation time per iteration, and solution is obtained rapidly. Unlike NR, fast decoupled load flow method is less sensitive to the initial voltage and reactive power conditions [SHI 06].

## **2-2 Problem formulation of OPF**

### **2-2.1 Background and definitions**

The OPF problem and the need for its solution were recognized in the late 1950s shortly after digital power flow programs became practical. The problem has been defined in several different ways, which led to misunderstanding about the nature of the problem and the success in solving it [TIN 87].

It is a type of a multicriteria optimization, which was defined by Carlos [COE 02] as the problem of finding “a vector of decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions. These functions form a mathematical description of performance criteria, which are usually in conflict with each other. Hence, the term “optimizes” means finding such a solution, which would give the values of the objective function to the designer.

The solving of such multicriteria problems has received a continuous attention by researchers in various fields including engineering.

In general, most of the constraints and objective functions of equations (1.1)-(1.3) represent the operational constraints and economical aims, respectively.

Optimizations of variables of these functions are either continuous or discrete. Continuous variables are dependable variables like voltage magnitude and voltage angle while the discrete ones are control variables such as active and reactive power of generation units.

Glover and Sarma [GLO 02] defined the Optimal Power Flow as a combination of economic dispatch with the power flow to solve the problem of optimizing the generation while enforcing the transmission network constraints.

## **2-2.2 OPF Objectives**

Before coming up with a detailed mathematical formulation of OPF, it is useful to address the objectives that the OPF is aiming to accomplish. This section will present some of the OPF goals as found in various literatures.

The OPF uses all control variables to help minimize the total costs of the power system operation.

An OPF study brings the following benefits:

- Cost saving due to reduced system losses
- Improved voltage control
- Improved system security; greater reserve margins
- Improved interchange transfer capabilities

OPF has been widely used in power system operation and planning.

The OPF determines system marginal cost data that, according to [WEB 95], can aid in the pricing of MW transactions as well as the pricing ancillary services such as voltage support through MVAR support.

From the discussion above, one can say OPF has a unique feature, which is to achieve an economical steady-state operation of the system while considering not only normal operating limits, but also violations that would occur during contingencies.

## **2-2.3 Objective Function**

The objective function can be any meaningful scalar function of the variables of the problem. The standard optimal power flow problem is the cost minimization. Cost minimization means minimizing the total cost of real power generation. Therefore, the objective function is the function of real power generation.

According to [BOU], generation cost, in \$/hr, of a generating plant  $i$  is represented by quadratic curves of second order:

$$C_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad (1.6)$$

where,

$P_{Gi}$  is the amount of generation in megawatts at generator  $i$ .

$a_i$ ,  $b_i$  and  $c_i$  are the unit costs curve for generator  $i$ .

The unit costs curve  $a_i$ ,  $b_i$  and  $c_i$  are in units of \$/hr, \$/MW.hr and \$/MW<sup>2</sup> .hr, respectively.

The objective function for the entire power system expressed as the sum of the quadratic cost model at each power plant is:

$$C_i = \sum_{i=1}^N (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \quad (1.7)$$

where,

$N$  is the number of generation including the slack bus.

The slack bus is a generator bus where the voltage magnitude and angle are specified.

### 2-2.4 Equality Constraints

While minimizing equation (1.6), it is very useful and critical for security reason, to ensure that the power system is operating in a normal condition i.e, power demand (load and losses) is satisfied and the network components are operating within limits. This can be achieved by active and reactive power transfer analysis. Thus, active and reactive power flow equations can be used as equality constraints.

$$P_G - P_{load} - P_{losses} = 0 \quad (1.8)$$

$$Q_G - Q_{load} - Q_{losses} = 0 \quad (1.9)$$

where,

$P_G$  and  $Q_G$  are generator active and reactive power outputs, respectively.

$P_{load}$  and  $Q_{load}$  are active and reactive load powers, respectively.

$P_{losses}$  and  $Q_{losses}$  are active and reactive transmission losses, respectively.

In summary, we can say, demand (load and losses) must equate to generated power. If the above equations are true, then the load demand (active and reactive power) is satisfied. This implies that the equality constraints are satisfied.

### **2-2.5 Inequality Constraints**

It is a well-known fact that most, if not all, of power system components and devices have limits or operating limits; these include limits created for system-security reason. Thus, the objective function can be minimized while ensuring that the network components are operating within limits. This brings in the concept of inequality conditions.

Voltage regulation requires bus-voltages to vary within certain limit, for example voltage values between 0.95 and 1.05 per unit are acceptable in most cases. Similarly, bus-voltage phase angles have upper and lower limits. Voltage and angle deviations at each bus are thus some of the essential inequality constraints of power system:

$$V^m \leq V \leq V^M \quad (1.10)$$

$$\delta^m \leq \delta \leq \delta^M \quad (1.11)$$

where,

$V$  is a vector of bus voltage magnitude with lower and upper limits,  $V^m$  and  $V^M$ , respectively.

$\delta$  is a vector of bus voltage phase angle with lower and upper limits,  $\delta^m$  and  $\delta^M$ , respectively.

### **2-2.6 Control Variables**

The control variables in an optimal power flow problem are quantities whose values can be adjusted directly to help minimize the objective function and satisfy the constraints.

Examples of optimal power flow control quantities are:

- Active power generation
- Reactive power generation

- Transformer tap ratios
- Generator bus voltages

Different classes of optimal power flow problems restrict the quantities that can be controlled. For example, an OPF algorithm for minimizing active power generation cost might limit the controls to active power generation [TIN 87].

### **Conclusion**

In this chapter, we have talked about the optimization problem and some of its applications in the engineering field. We have also spoken about the optimization on the power flow. In the coming chapter, we will see some methods used for the optimization and among those methods; we will speak intensely about the GA (Genetic Algorithm) and the PSO (Particle Swarm Optimization) techniques.

# **Chapter 2**

## **Optimization Methods**

## **Introduction**

Optimization methods are widely used in various fields, including engineering, economics, management, physical sciences, social sciences, etc. The task is to choose the best or a satisfactory one from amongst the feasible solutions to an optimization problem, providing the scientific basis of decision-making for decision makers [JUN 12].

In this present chapter, we try to talk briefly about some methods used for the optimization, here we mean, conventional (traditional) methods and non conventional (evolutionary) methods, among those; we intend to speak intensely about the GA (Genetic Algorithm) and the PSO (Particle Swarm Optimization) techniques.

### **1- Optimization methods classification**

#### **1-1 Traditional Algorithms**

This section presents some conventional/traditional methods of power system optimization as found in various literatures.

The optimal power flow is a very large and difficult mathematical programming problem [WOO 84]. Since its introduction, almost every mathematical programming approach has been attempted to solve OPF problem.

It has taken researchers many decades to develop computer codes that will solve the OPF problem reliably [WOO 84]. Conventional methods such as Newton-based, Lambda iteration and Gradient methods came out of operational researches and have been used since then with some success.

Each of these methods has advantages and disadvantages as discussed below. [WOO 84]

Newton-based method converges rapidly in many cases where other methods diverge. However, this method gives problems with inequality constraints.

In lambda iteration method, the penalty factors may be calculated outside by a power flow. Lambda iteration method forms the basis of many standard on-line economic dispatch programs.

Gradient methods are slow in convergence and are difficult to solve in the presence of inequality constraints.

Since the convergence is of great essence in most systems, the application of Newton-based method in power system is extensive. A brief discussion of Newton method is given below.

### 1-1.1 Newton based algorithm

Newton's method has been the standard solution algorithm for the power flow problem for decade. Newton's method is a very powerful solution algorithm because of its rapid convergence near the solution. This property is especially useful for power system applications because an initial guess near the solution is easily attained. System voltages will be near rated system values, generator outputs can be estimated from historical data, and transformer tap ratios will be near 1.0 p.u [WEB 95].

Application of Newton to optimization problem is as follows:

$$\left\{ \begin{array}{ll} \text{Minimize,} & f(x) \end{array} \right. \quad (2.1)$$

$$\left\{ \begin{array}{ll} \text{Subject to:} & g(x) = 0 \end{array} \right. \quad (2.2)$$

$$h(x) \leq 0 \quad (2.3)$$

where,

$f$  : objective function

$g$  : equality constraints

$h$  : inequality constraints

The solution of this problem by Newton's method requires the creation of the Lagrangian,

$$L(z) = f(x) + \lambda^T h(x) + \mu^T h(x) \quad (2.4)$$

where,

$z = [x \ \mu \ \lambda]^T$ ,  $\mu$  and  $\lambda$  are vectors of the Lagrange multipliers, and  $h(x)$  only includes the active (or binding) inequality constraints.

Once an understanding of the calculation of the Hessian and gradient is attained, the solution of many optimization problems can be achieved by using the Newton's method algorithm.

### **1-1.2 Problems with traditional approaches**

It was found that most traditional approaches of solving this nonlinear optimization have limitations especially in modeling of nonconvex objective functions, discrete control variables, and prohibited unit operating zones [BAK 02].

In addition, most conventional methods are either too slow or too weak to be of any practical value [BOU].

It was stated in [BOU] that applications of conventional optimization techniques to large system with a very non-linear objective functions and great number of constraints are not good enough. This is because most conventional methods depends on the existence of the first and the second derivatives of the objective function and on the well computing of these derivative in large search space. This is one of weak spots of these methods, because the generalized objective function is a non-linear and the number of constraints increases with the size of system under consideration.

Due to these limitations, many other powerful deterministic, probabilistic and stochastic techniques for solving large dimensional optimization problems were proposed.

The most popular types are the Evolutionary Algorithm (EA) based solution techniques; these are probabilistic algorithms, which have the ability of simultaneous multidimensional search for optimal solution.

### **1-2 Evolutionary Algorithms (EAs)**

The EAs are search methods that take their inspiration from natural selection and survival of the fittest in the biological world. They differ from more traditional optimization techniques in that they involve a search from a "population" of solutions, not from a single point [14]. Most discussions that follow are based on this reference.

EAs perform well on noisy functions where there may be multiple local optima. They tend not to get stuck on local minima and can often find globally optimal solutions. EAs are well suited for a wide range of combinatorial and continuous problems.

EAs model natural processes, such as selection, recombination and mutation.

Selection determines which individuals are chosen for mating and how many offspring each selected individual produces.

Recombination produces new individuals in combining the information contained in the parents.

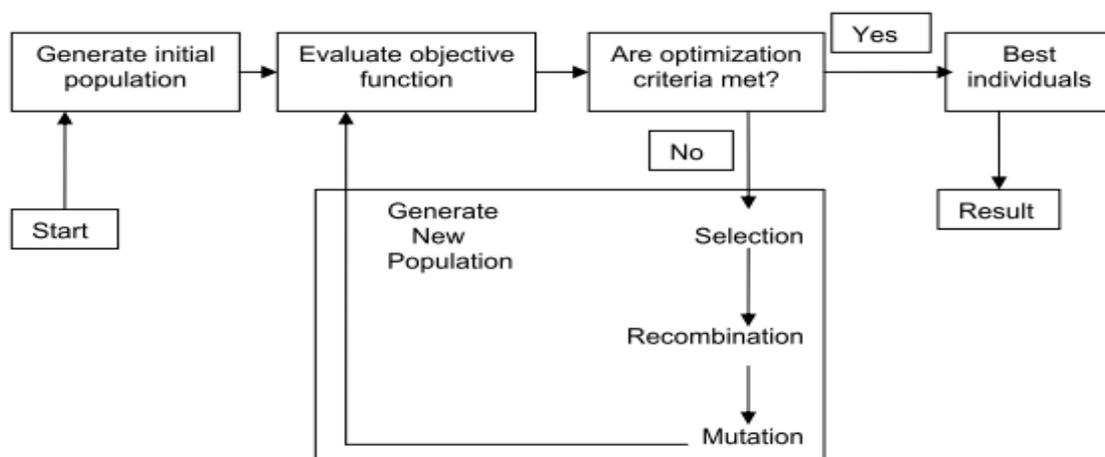
After recombination every offspring undergoes mutation. Offspring variables are mutated by small size of the mutation step, with low probability.

At the beginning of the computation, a number of individuals are randomly initialized. The objective function is then evaluated for these individuals. The initial generation is produced. If the optimization criteria are not met the creation of a new generation starts. Individuals are selected according to their fitness for the production of offspring. Parents are recombined to produce offspring. All offspring will be mutated with a certain probability. The fitness of the offspring is then computed. This cycle is performed until the optimization criteria are reached.

(Figure 2.1) shows the structure of a simple evolutionary algorithm [14]. The reference has highlighted the following as the most significant differences between evolutionary algorithms (EAs) and conventional search are:

- EAs search a population of points in parallel, not just a single point
- EAs do not require derivative information
- EAs use probabilistic transition rules, not deterministic ones
- EAs can provide a number of potential solutions to a given problem.

For these reasons, the EAs found its application in power system to solve optimization problem of OPF.



**Figure 2.1:** Structure of a single population evolutionary algorithm.

## **2- Genetic Algorithms (GAs)**

### **2-1 Overview of the GA**

The genetic algorithm is a method for solving both constrained and unconstrained optimization problems that is based on natural selection.

Genetic algorithms are resolution algorithms based on the mechanics of natural selection and natural genetics [OUI 05]. They combine survival of the fittest among string structures to form a resolution algorithm with some of man's capacity for survival. In every generation, a new set of artificial strings is created by using bits and pieces from the fittest of the old; genetic algorithms are no simple random walk, they efficiently exploit historical information to speculate on new research points with expected improved performance.

### **2-2 GAs operators**

Genetic Algorithm is an iterative procedure, which maintains a constant size population **P** of candidate solutions. A population is a set of points in the search space. During each iteration step; three genetic operators: reproduction, crossover and mutation are performing to generate new populations and the chromosomes of the new populations are evaluated via the value of the fitness, which is related to objective function.

- Reproduction is based on the principle of survival of the better fitness. It obtains a fixed number of copies of solutions according to their fitness value.
- Crossover promotes the exploration of new regions in the search space. It combines two parents to form children for the next generation.
- Mutation prevents premature stopping of the algorithm in a local solution. This operator is defined by a random bit value change in a chosen string with a low probability.

Based on these genetic operators and the evaluations, the better new populations of candidate solution are formed. Over successive generations, the population progresses toward an optimal solution.

The method is not sensitive to the starting points and is capable to determining the global optimum solution to the optimization problem for a range of constraints and objective functions. Genetic Algorithm can be applied to solve difficult optimization problems with objective functions that do not possess properties such as continuity, differentiability, etc.

With the previous description, a simple genetic algorithm is outlined as follows [BOU]:

**Step 1:** Generate randomly a population string

**Step 2:** Calculate the fitness for each string in the population

**Step 3:** Create offspring through genetic operators

**Step 4:** Evaluate the strings and calculate the fitness for each string

**Step 5:** If the search goal is achieved, return the best string as the solution; otherwise go to step 3.

### 2-3 GAs parameters

According to [JAC 05], the genetic algorithms are effectively derived from a simple model of population genetics and have the following parameters: *reproduction*, *population size*, *crossover* and *mutation*.

*Reproduction* is a process by which individual strings are copied according to their fitness values. Copying strings according to their fitness values means that strings with higher values have a higher probability of contributing one or more offspring in the next generation.

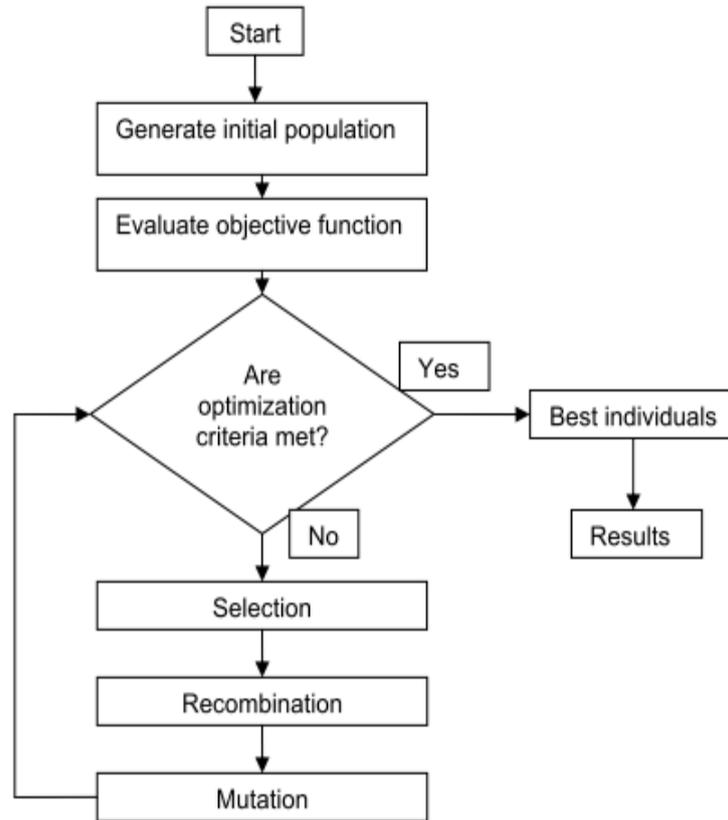
*Population* represents the states in GA. The larger the population the more accuracy GA will be able to infer the next state.

*Crossover* is the structured recombination operation by exchanging genes or parameters.

Crossover rate allows the exploration of solution space around the parent solution.

*Mutation* is the random alternation of the genes. Mutation rate controls the rate when new genes are introduced and explores new solution territory.

Simple Genetic Algorithm includes all the parameters discussed above. The flow chart of simple Genetic Algorithm is shown in (Figure 2.2). [JAC 05]



**Figure 2.2:** Flow chart of a simple Genetic Algorithm.

The recombination operation used by GAs requires that the problem can be represented in a manner that makes combinations of two solutions likely to generate interesting solutions. Consequently selecting an appropriate representation is a challenging aspect of applying this method a website .

Currently GAs are the most popular type of evolution algorithms and there has been a good deal of interest in applying them.

The main differences between GAs and conventional search are [BOU]:

- GAs work with a population of strings, searching many peaks in parallel, as opposed to a single point.
- GAs work directly with strings of characters representing the parameters set the parameters themselves.
- GAs use probabilistic transition rules instead of deterministic rules

- GAs do not require derivative information, but use objective function information
- GAs have the potential to find solutions in many different areas of the search space simultaneously.

When applying GAs to a particular optimization problem (OPF in this case), two main issues must be addressed. According to [BAK 02], these are:

- The encoding and its inverse operator, decoding;
- The definition of the fitness function to be minimized or maximized by the GA

Encoding is about translating the problem physical variables to a GA chromosome. This is required because GA works with a population of chromosome, not with parameters themselves.

The fitness function is a measure of the quality of each candidate solution. It is the function that needs to be optimized i.e. the objective function. Some optimization problems are to minimize the objective function. Thus, the definition of the fitness function is required because GA is designed to either maximize or minimize the fitness function.

### **3- Particle Swarm Optimization (PSO)**

#### **3-1 Overview of the PSO**

Particle Swarm Optimization Swarm Intelligence (SI) is an innovative distributed intelligent paradigm for solving optimization problems that originally took its inspiration from the biological examples by swarming, flocking and herding phenomena in vertebrates. Particle Swarm Optimization (PSO) incorporates swarming behaviors observed in flocks of birds, schools of fish, or swarms of bees, and even human social behavior, from which the idea is emerged.

PSO is a population-based optimization tool, which could be implemented and applied easily to solve various function optimization problems. As an algorithm, the main strength of PSO is its fast convergence, which compares favorably with many global optimization algorithms like Genetic Algorithms (GA) Simulated Annealing (SA) and other global optimization algorithms. For applying PSO successfully, one of the key issues is finding how to map the problem solution into the PSO particle, which directly affects its feasibility and performance Similar to evolutionary algorithm, the PSO technique conducts

searches using a population of particles, corresponding to individuals. Each particle represents a candidate solution to the optimal power flow problem. In a PSO system, particles change their positions by flying around in a multidimensional search space until a relatively unchanged position has been encountered, or until computational limitations are exceeded. In social science context, a PSO system combines a social –only model and a cognition-only model. The social-only component suggests that individuals ignore their own experience and adjust their behavior according to the successful beliefs of the individual in the neighborhood. On the other hand, the cognition-only component treats individuals as isolated beings. A particle changes its position using these models [BHU 13].

### **3-2 Basic elements of the PSO**

The basic elements of the PSO techniques are briefly stated and defined as follow [BHU 13]:

**Particle  $X(t)$ :** It is a candidate solution represented by an m-dimensional real valued vector, where m is the number of optimized parameters. At time t, the  $i^{\text{th}}$  particle  $X_i(t)$  can be described as  $X_i(t)=[x_{i,1}(t) ; x_{i,2}(t) ; \dots ; x_{i,m}(t)]$ .

**Population:** it is a set of n particles at time t, i.e  $\text{pop}(t) = [X_1(t), X_2(t), \dots, X_n(t)]^T$ .

**Swarm:** it is an apparently disorganized population of moving particles that tend to cluster together while each particle seems to be moving in a random direction.

**Inertia weight  $w(t)$ :** it is a control parameter that is used to control the impact of the previous velocity on the current velocity. All the control variables transformer tap positions and switch-able shunt capacitor banks are integer variables and not continuous variables. Therefore, the value of the inertia weight is considered to be 1 in this study.

**Individual best  $X^*(t)$ :** As the particle moves through the search space, it compares its fitness value at the current position to the best fitness value it has ever attained at any time up to the current time. The best position that is associated with the best fitness encountered so far is called the individual best  $X^*(t)$ . For each particle in the swarm,

$X^*(t)$  can be determined and updated during the search.

**Global best  $X^{**}(t)$ :** It is the best position among all of the individual best positions achieved so far.

**Stopping criteria:** These are the conditions under which the search process will terminate. In this study, the search will terminate if one of the following criteria is satisfied:

The number of the iterations since the last change of the best solution is greater than a pre-specified number.

The number of iterations reaches the maximum allowable number.

### 3-3 PSO algorithm

In a PSO algorithm, the population has  $n$  particles that represent candidate solutions. Each particle is a  $k$ -dimensional real-valued vector, where  $k$  is the number of the optimized parameters. Therefore, each optimized parameter represents a dimension of the problem space. The modified PSO technique for integer problem can be described in the following steps [BHU 13].

**Step 1:** (Initialization): Set  $t=0$  and generate random  $n$  particles,  $\{X_i(0), i=1,2,..n\}$ . Each article is considered to be solution for the problem and it can be described as  $X_i(0)=[x_{i,1}(0); x_{i,2}(0); \dots; x_{i,m}(0)]$ . Each control variable will have a range  $[x_{min}, x_{max}]$ . Each particle in the initial population is evaluated using the objective function  $f$ . For each particle, set  $X_i^*(0) = X_i(0)$  and  $f_i^* = f_i, i=1,2,3,\dots,n$ . Search for the best value of the objective function  $f_{best}$ . Set the particle associated with  $f_{best}$  as the global best,  $X^{**}(0)$ , with an objective function of  $f^{**}$ . Set the initial value of the inertia weight  $w(0)$ . In this study the objective function is the optimal power flow, which will be calculated after running the power flow and meeting all our constraints.

**Step 2:** Counter Updating: update the counter  $t = t + 1$ .

**Step 3:** Velocity updating: Using the global best and individual best, the  $i$ th particle velocity in the  $k$ th dimension in this study (integer problem) is updated according to the following equation:  $v_{i,k}(t) = w(t).v_{i,k}(t-1) + b_1s_1(x_{i,k}^*(t-1) - x_{i,k}(t-1)) + b_2s_2(x^{**}_{i,k}(t-1) - x_{i,k}(t-1))$

From the previous equation  $i$  is the particle number,  $b_1$  and  $b_2$  are positive constants,  $s_1$  and  $s_2$  are uniformly distributed random numbers in  $[0, 1]$  and  $k$  is the  $k$ th control variable. Then, check the velocity limits. If the velocity violated its limit, set it at its proper limit. The second term of the above equation represents the cognitive part of the PSO where the particle changes its velocity based on its own thinking and memory.

The third term represents the social part of PSO where the particle changes its velocity based on the social-psychological adaptation of knowledge.

**Step 4:** Position updating: Based on the updated velocity, each particle changes its position according to the following equation:  $X_{i,k}(t) = x_{i,k}(t-1) + v_{i,k}(t)$

**Step 5:** Individual best updating: each particle is evaluated and updated according to the update position.

**Step 6:** Search for the minimum value in the individual best and its solution has ever been reached so far, and consider it to be the minimum.

**Step 7:** Stopping criteria: if one of the stopping criteria is satisfied, then stop otherwise go to step-2.

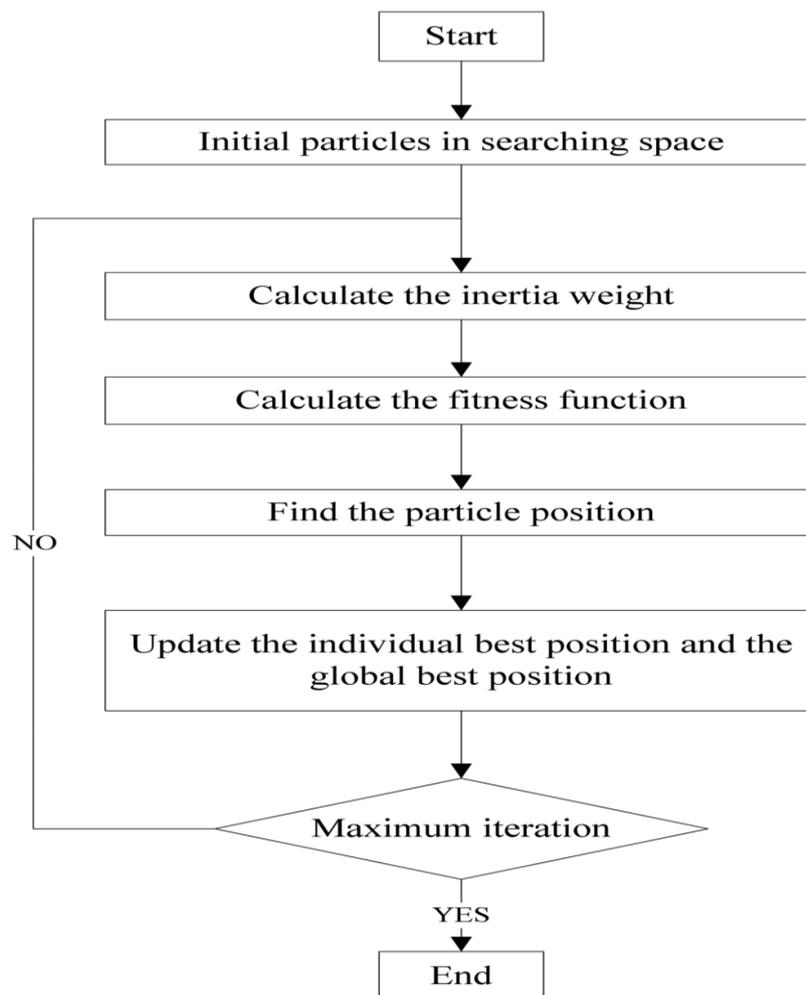


Figure 2.3: Flow chart Of PSO.

### **3-4 Advantages of PSO**

The advantages of PSO over other traditional optimization techniques can be summarized as follow [BHU 13]:

- a)** PSO is a population-based search algorithm (i.e., PSO has implicit parallelism). This property ensures PSO to be less susceptible to getting trapped on local minima.
- b)** PSO uses payoff (performance index or objective function) information to guide the search in the problem space. Therefore, PSO can easily deal with non-differentiable objective functions. Additionally, this property relieves PSO of assumptions and approximations, which are often required by traditional optimization models.
- c)** Stochastic optimization algorithm that can search a complicated and uncertain area. This makes PSO more flexible and roust than conventional methods.
- d)** Unlike Genetic Algorithm (GA) and other heuristic algorithms, PSO has the flexibility to control the balance between the global and local exploration of the search space. This unique feature of a PSO overcomes the premature convergence problem and enhances the search capability.
- e)** Unlike the traditional methods, the solution quality of the proposed approach doesn't rely on the initial population.

Starting anywhere in the search space, the algorithm ensures the convergence to the optimal solution.

### **Conclusion**

In this second chapter, we talked briefly about some methods used for the optimization, conventional (traditional) methods and non conventional (evolutionary) methods, among those; we spoke intensely about the GA (Genetic Algorithm) and the PSO (Particle Swarm Optimization) techniques. In the coming chapter, we will try to make a comparative analysis between the two methods concerning their efficiency on the power system by applying them on a 30 Bus grid.

# **Chapter 3**

## **Comparative Analysis**

## Introduction

This chapter is concerned with making a comparative analysis between two evolutionary methods: GAs and PSO, by applying them on a grid of 30 Bus to see their efficiency on the system. But before that, we will try to deal with two main functions which are cost function (economic dispatching) and also emission function (environmental dispatching).

The using approach for the application will be MATLAB software. So, MATLAB (Matrix Laboratory), a product of MathWorks, is a scientific software package designed to provide integrated numeric computation and graphics visualization in high-level programming language. Dr Cleve Moler, Chief scientist at MathWorks, Inc., originally wrote MATLAB, to provide easy access to matrix soft-ware. The very first version was written in the late 1970s for use in courses in matrix theory, linear algebra, and numerical analysis. MATLAB is therefore built upon a foundation of sophisticated matrix software, in which the basic data element is a matrix that does not require predimensioning [DEE 08].

### 1- Choice of the objective functions to be optimized

#### 1-1 Cost function

It is also known by the economic dispatching, this function reflects the need to minimize the total cost of production of the active powers. It is assumed that the individual cost of each production center only depends on the generation of active power [SAS 69].

$$F = \sum_{i=1}^{ng} f_i = \sum_{i=1}^{ng} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad (3.1)$$

where,  $n_g$  : is the number of generators and  $P_{Gi}$  is the generated active power at Bus  $i$ .

$a_i, b_i, c_i$  : coefficients of the generated power for generator  $i$ .

#### 1-1.1 Economic dispatching

To ensure the continuity of electricity supplying to customers, all modern transmission grids are interconnected where several producers (power plants) are interlinked, but the management of all these plants required the creation of a coordination tool called the dispatcher, as the name indicated, this dispatcher is used to distribute the total power generated.

The economic dispatching returns to find the total power distribution between all power plants supplying this network to satisfy the power demand of customers more power loss in the lines provided that this distribution should primarily supply electricity with minimum production cost. The cost of producing each unit is represented by a proper quadratic function to this plant whose form is:

$$Fc = a + b_i P_{Gi} + c_i P_{Gi}^2 \quad (3.2)$$

where a, b and c are measured coefficients.

### 1-1.2 Dispatching problem formulation

$$\left\{ \begin{array}{l} \text{Min} \quad Fc = a + b_i P_{Gi} + c_i P_{Gi}^2 \\ \text{Subject to:} \\ g(x) = 0 \quad (\text{equality constraints}) \\ P_{Gi} - P_{di} - \text{Losses} = 0 \quad (3.3) \\ h_i(x) \leq 0 \quad (\text{inequality constraints}) \\ P_{G \min} \leq P_G \leq P_{G \max} \quad (3.4) \\ Q_{G \min} \leq Q_G \leq Q_{G \max} \quad (3.5) \\ V_{i \min} \leq V_i \leq V_{i \max} \quad (3.6) \\ S_{Tr \ ij} \leq S_{Tr \ ij \ max} \quad (3.7) \end{array} \right.$$

where,

$F_C$  : Optimized cost function.

$P_{Gi}$  : Generated active power at Bus i,  $P_{di}$ : Demanded power at Bus i.

$P_{Gmax}$  : Maximal generated active power in the grid,  $P_{Gmin}$  : Minimal generated active power in the grid.

$V_{imax}$  : Maximal tension at Bus i,  $V_{imin}$  : Minimal tension at Bus i.

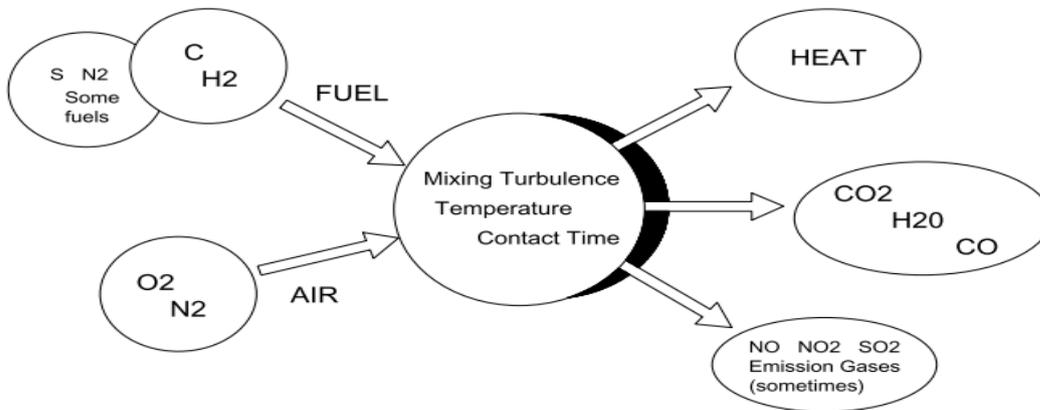
$Q_{Gmax}$  : Maximal generated reactive power in the grid,  $Q_{Gmin}$  : Minimal generated reactive power in the grid.

$S_{Tr \ ij}$  : Transited power,  $S_{Tr \ ij \ max}$  : Maximal transited power.

### 1-2 Emission function

Production of electricity from conventional plants is causing combustion where the combination of the oxygen in air and the carbon forming the carbon dioxide (CO<sub>2</sub>) and generating heat is a complex process where high concentrations of undesirable products may be formed. Carbon monoxide (CO), for example, is the result of poor mixing of fuel and air or too little air. Other side products such as nitrogen oxides (NO, NO<sub>2</sub>) are formed in excessive amounts when the burner flame temperature is too high. If a fuel containing sulfur, sulfur dioxide (SO<sub>2</sub>) gas is formed.

And from (Figure 3.1), any burning is accompanied by emission of toxic gases.



**Figure 3.1:** Diagram of Combustion.

This emission function is used to calculate the amount of toxic gas emitted by the different power generation units composing the grid; it is mainly based on power generated from these units [YAL 07].

Minimization of: 
$$F(P_{G_i}) = \sum_{i=1}^{n_g} 10^{-2}(\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2) + \zeta_i \exp(\lambda_i P_{G_i}) \quad (3.8)$$

Where,  $n_g$  : is the number of generators and  $P_{G_i}$  is the generated active power at Bus  $i$ .

$\alpha_i, \beta_i, \gamma_i, \lambda_i$  : coefficients of the generated power for generator  $i$ .

#### 1-2.1 Environmental dispatching

To remedy the problem of the atmospheric pollution caused primarily by the emission of toxic gases from power plants based on hydrocarbons, the replacement of these conventional

sources of power from renewable sources presents an ideal solution but in reality it is a bit utopian.

Another alternative just won since it does not require additional investment, this solution is like looking for a distribution of generated powers between different power plants supplying the grid so that these power plants emit the minimum gas toxic.

The amount of toxic gases from each station is represented by a specific function to each plant whose expression is:

$$F_E = \sum_{i=1}^{n_g} 10^{-2}(\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2) + \zeta_i \exp(\lambda_i P_{G_i})$$

where,  $n_g$  : is the number of generators and  $P_{G_i}$  is the generated active power at Bus i.

$\alpha_i, \beta_i, \gamma_i, \lambda_i$  : coefficients of the generated power for generator i.

### 1-2.2 Optimization problem formulation of environmental dispatching

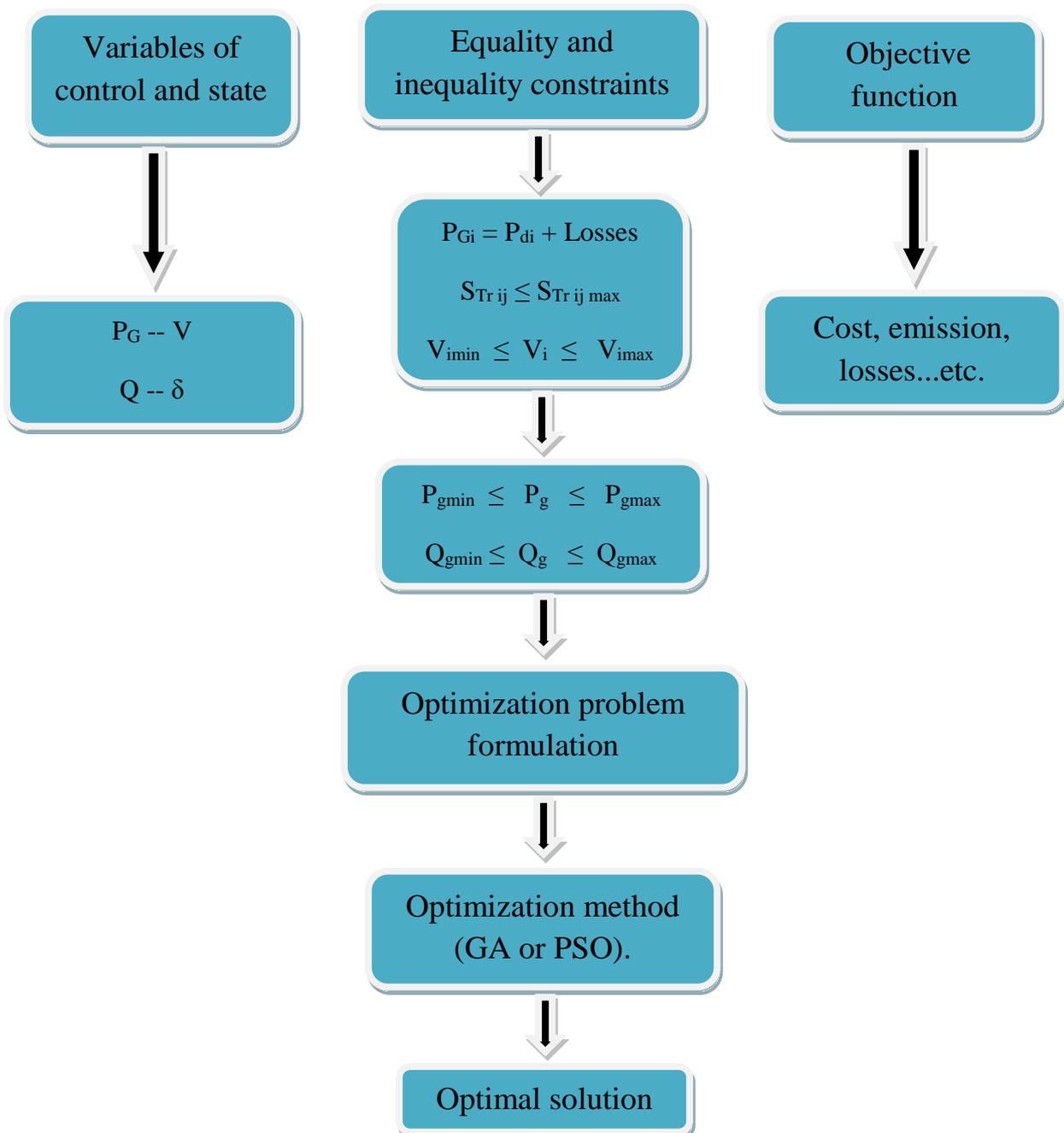
$$\left\{ \begin{array}{l} \text{Min} \quad F_E(P_{G_i}) = \sum_{i=1}^{n_g} 10^{-2}(\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2) + \zeta_i \exp(\lambda_i P_{G_i}) \\ \text{Subject to:} \\ g(x) = 0 \quad (\text{equality constraints}) \\ \quad \quad \quad P_{G_i} - P_{d_i} - \text{Losses} = 0 \\ h_i(x) \leq 0 \quad (\text{inequality constraints}) \\ \quad \quad \quad P_{G_{\min}} \leq P_G \leq P_{G_{\max}} \\ \quad \quad \quad Q_{G_{\min}} \leq Q_G \leq Q_{G_{\max}} \\ \quad \quad \quad V_{i \min} \leq V_i \leq V_{i \max} \\ \quad \quad \quad S_{Tr \ ij} \leq S_{Tr \ ij \ max} \end{array} \right.$$

where,

$F_E$  : Optimized emission function.

**1-3 Required conditions forming an optimization problem.**

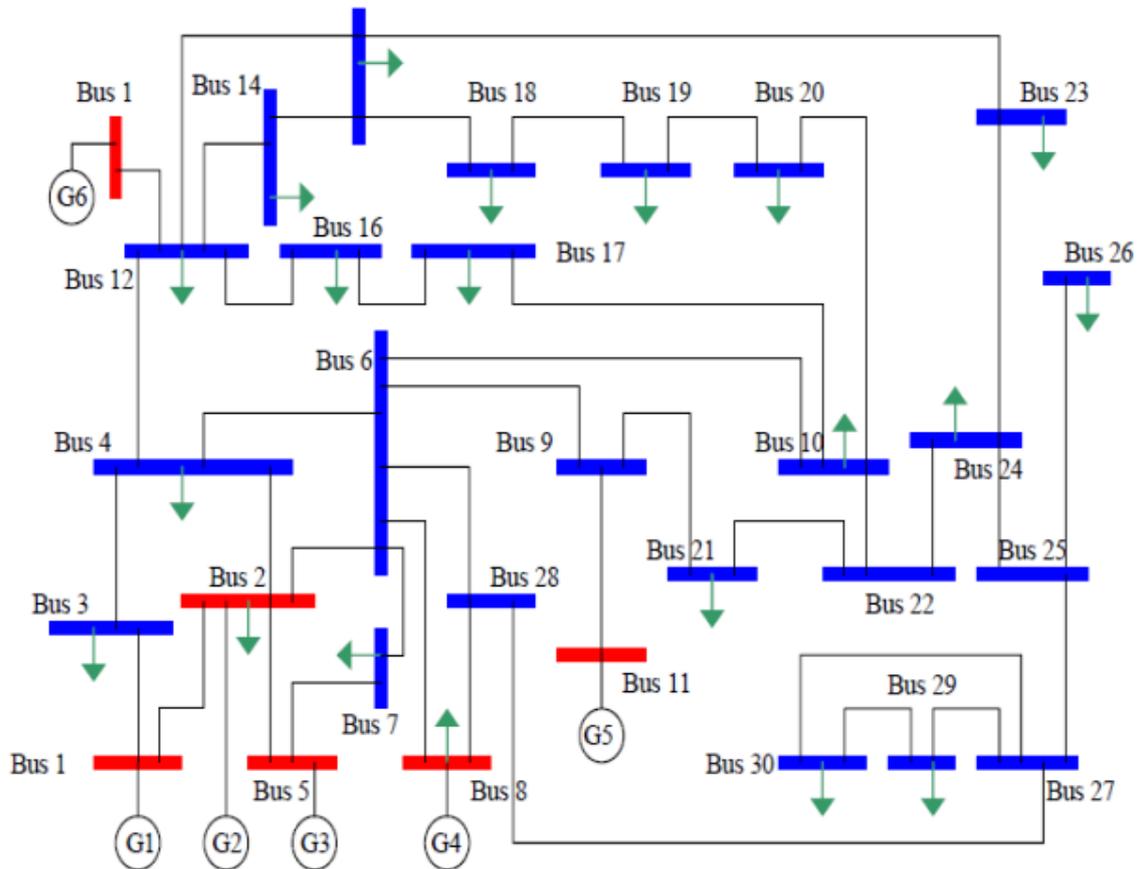
In the following figure (Figure 3.2), we will explain what the required conditions are in order to formulate an optimization problem and the possibility of obtaining an optimal solution [KHA 13].



**Figure 3.2:** Bloc diagram represents the required conditions to form an optimization problem.

## 2- Application

- 30 Bus power grid.



**Figure 3.3:** Single-line diagram of the IEEE 30 Bus power grid.

The grid data is shown in the Appendix.

### 2-1 GA Method

#### 2-1.1 Choice of parameters

- Population size: 100
- Number of generations: 20
- Number of lines: 41
- Crossover fraction: 0.75
- Selection type: roulette
- Crossover type: two points
- Mutation probability: 0.008.

**2-1.2 Cost function  $F_c$ :**

We start first with the generation cost function. After the optimization program execution with the previous indicated parameters, we obtained these coming results:

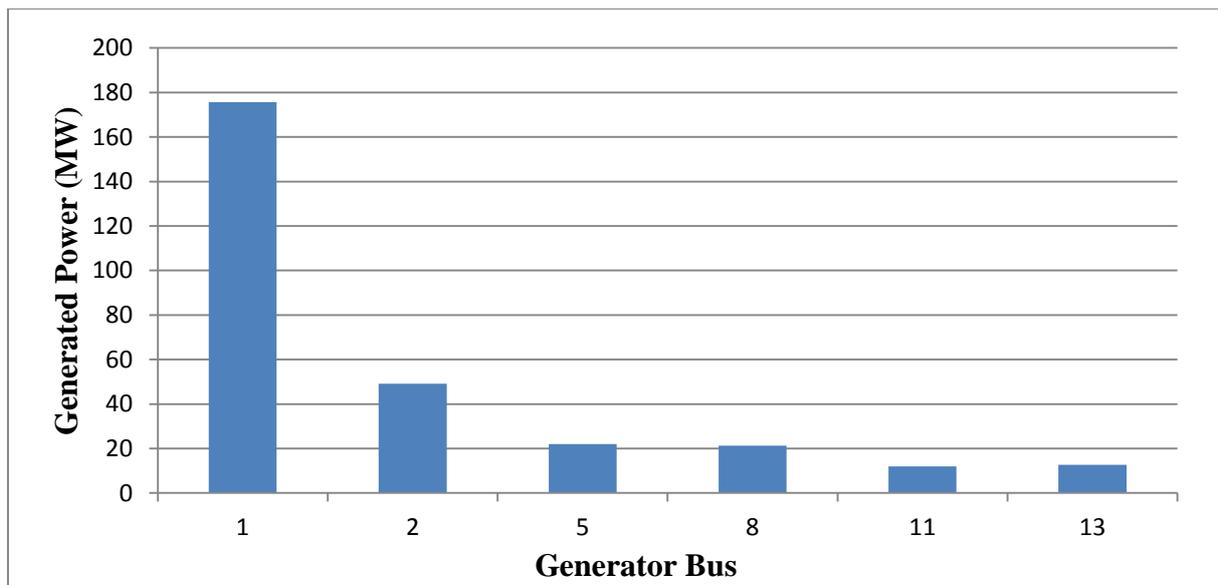
**Table 3.1:** Optimized cost function and non-optimized emission function values.

Optimized cost function value (Dollars/Hour)	Non-optimized emission function value (Kg/Hour).	Non-optimized losses function value (MW)
801.905	327.166	9.3290

And the generated power values corresponding to those three values are:

**Table 3.2:** Optimal repartition of generated powers.

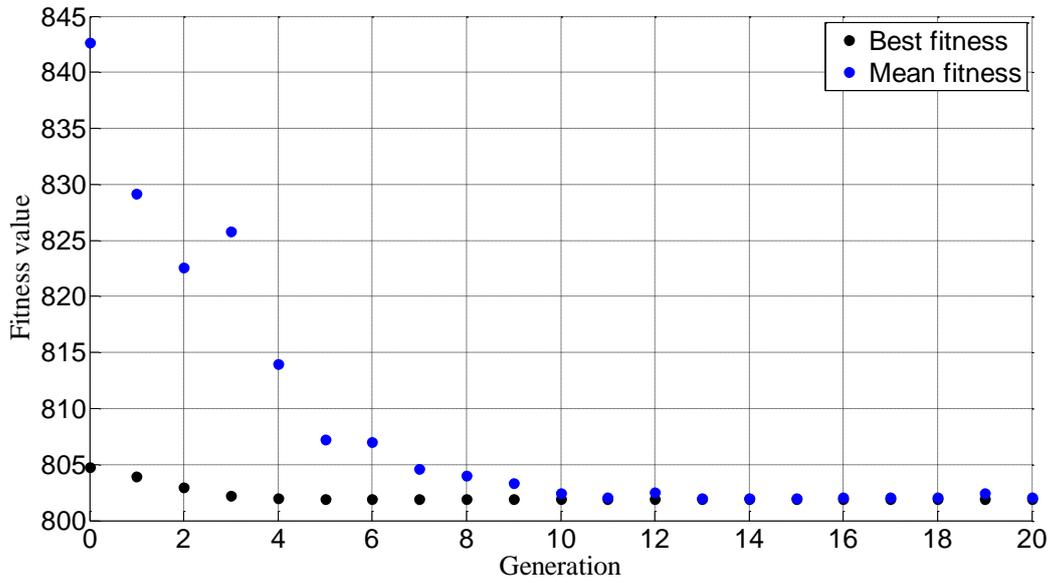
Optimal repartition of generated powers for each production unit	
Generator Bus (N°)	Generated Power (MW)
1	175.7391
2	49.1161
5	21.9960
8	21.2668
11	11.9577
13	12.6325



**Figure 3.4:** Optimal repartition of generated powers.

The calculation results are presented in the next figure:

Elapsed time is:  $t = 419.9598$  s

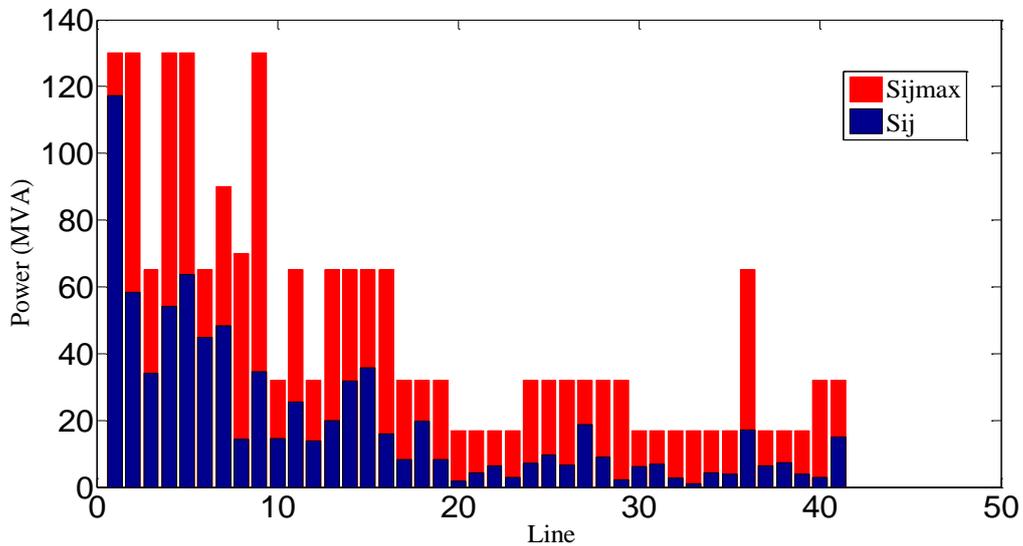


**Figure 3.5:** Evolution of the objective function over generations.

**Best fitness:** represents the best value of fitness function ( $\sum F_i / n$ ),  $n$ : solutions number.

**Mean fitness:** represents the mean value (minimal) of solutions.

### 2-1.2.1 Verification of constraints



**Figure 3.6:** Verification of transited powers constraint.

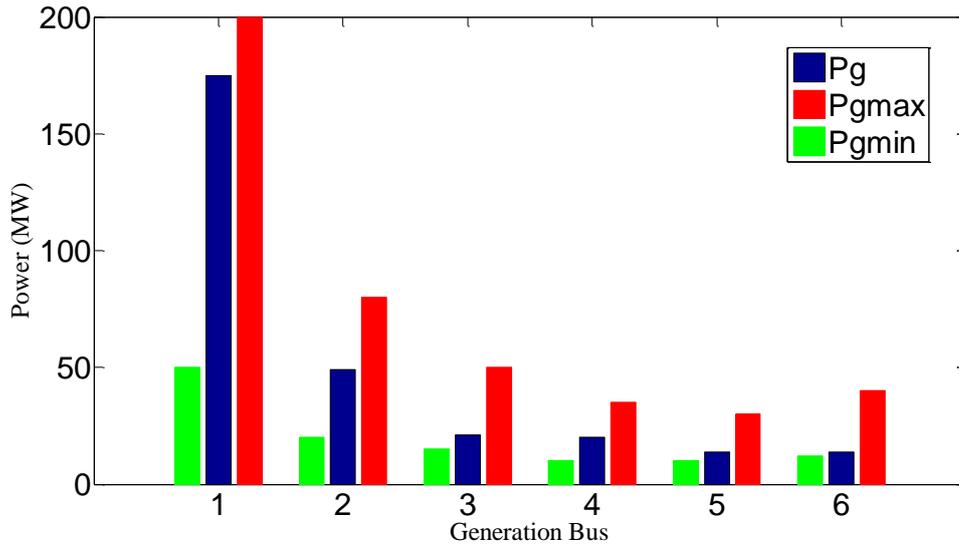


Figure 3.7: Verification of generated powers constraint.

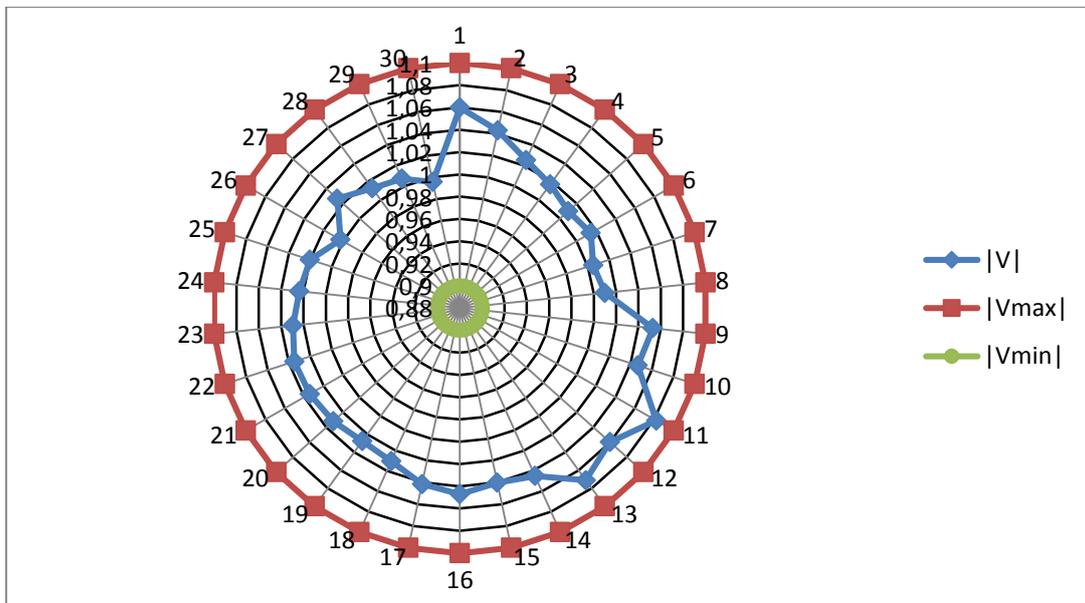


Figure 3.8: Verification of tensions constraint.

### 2-1.2.2 Discussion and Interpretation

The figure (3.4) shows the development of the optimal solution and the average cost solutions based on generations.

It is clear that the figure shows the reduction of  $F$  between the optimum value and the average value by generations, especially in the beginning and the frequent rebounding average value.

The initial sharp reduction proves that our algorithm works and as convergence, while the frequent rebound is due to the significant value of the mutation which causes a departure from the optimal solution for return.

This is the solution to avoid falling into a local minimum but by determining the speed of the algorithm.

According to and as for the figures (3.6), (3.7) and (3.8), we can say that the optimization results obtained by the genetic algorithm are calculated in accordance with the technical requirements imposed by the PF, in our grid where the voltages at all the buses are in the allowed range:

$$V_{i \min} \leq V_i \leq V_{i \max} \quad (3.9)$$

Same thing for the generated powers:

$$P_{g \min} \leq P_g \leq P_{g \max} \quad (3.10)$$

Transited powers in the figure are below their minimum value where the grid avoids the heating on the lines.

**Note:**

We must notice that the respect of the constraints imposed to the grid is ensured by the calculation of the PF for the studied cases in this thesis.

**2-1.3 Emission function F<sub>E</sub>:**

Now, we move to the generation emission function. After the optimization program execution with the previous indicated parameters, we obtained these coming results:

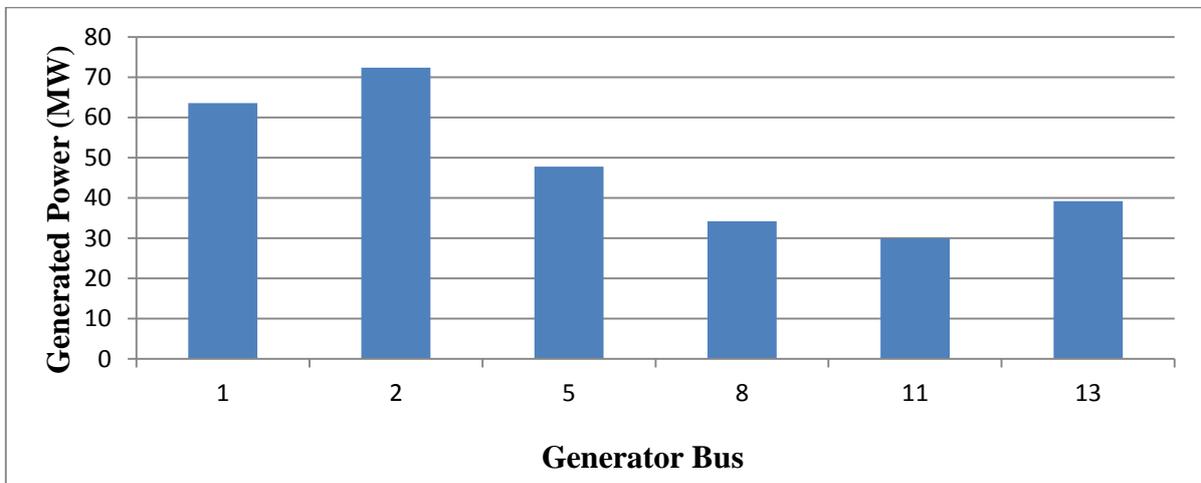
**Table 3.3:** Non-optimized cost function and optimized emission function values.

Non-optimized cost function value (Dollars/Hour)	Optimized emission function value (Kg/Hour).	Non-optimized losses function value (MW)
196.714	940.418	3.6618

And the generated power values corresponding to those three values are:

**Table 3.4:** Optimal repartition of generated powers.

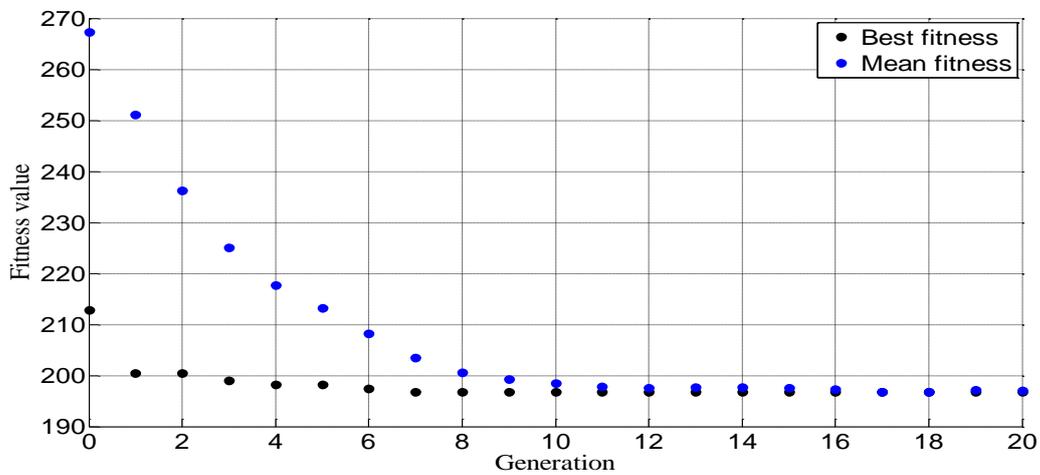
Optimal repartition of generated powers for each production unit	
Generator Bus (N°)	Generated Power (MW)
1	63.5943
2	72.3933
5	47.8121
8	34.1863
11	29.9217
13	39.2011



**Figure 3.9:** Optimal repartition of generated powers.

The calculation results are presented in the next figure:

Elapsed time is:  $t = 275.022631$  s



**Figure 3.10:** Evolution of the objective function over generations.

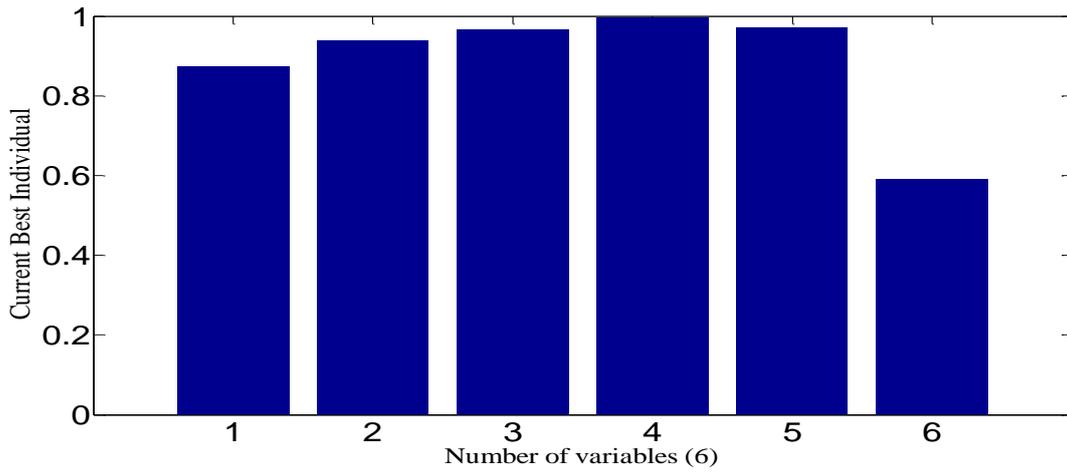


Figure 3.11: Current best individual.

2-1.3.1 Verification of constraints

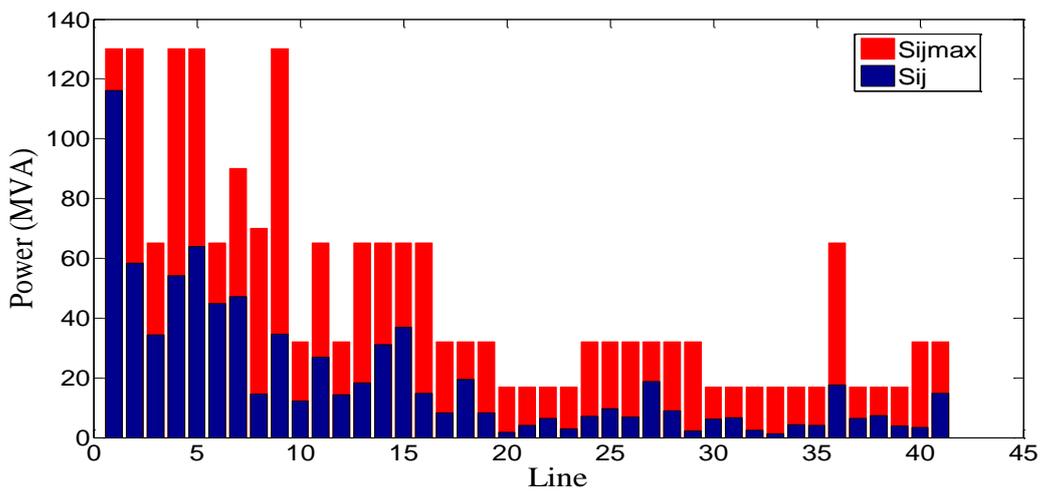


Figure 3.12: Verification of transited powers constraint.

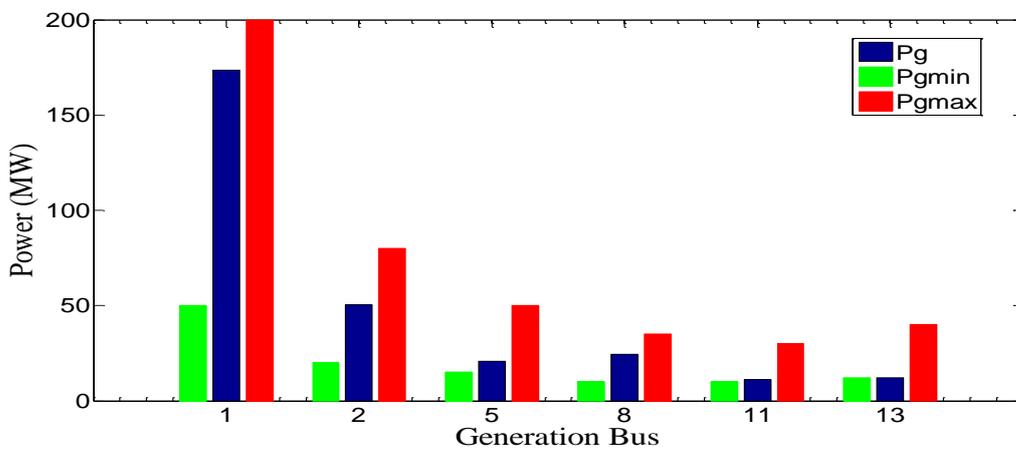
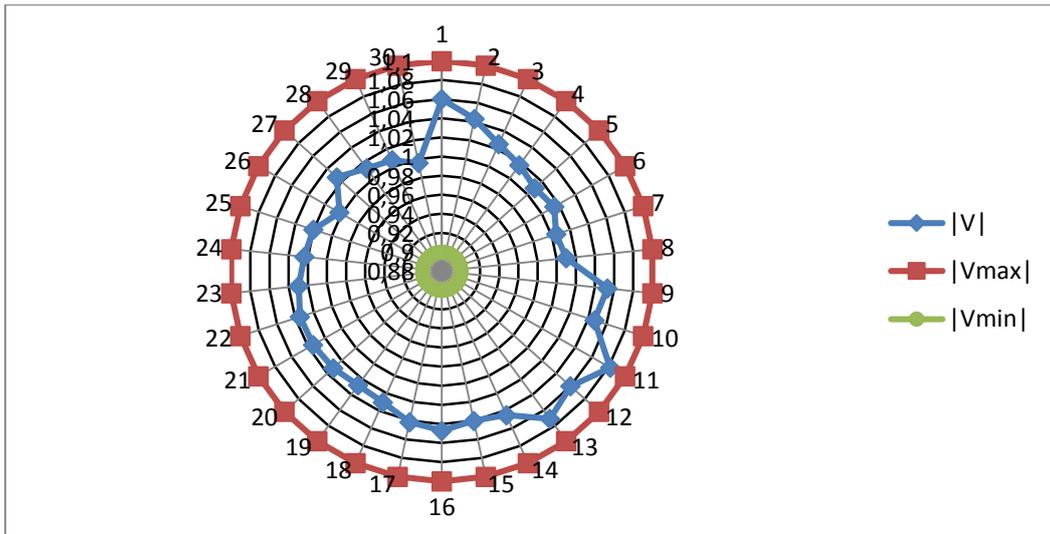


Figure 3.13: Verification of generated powers constraint.



**Figure 3.14:** Verification of tensions constraint.

### 2-1.3.2 Discussion and Interpretation

The figure (3.9) shows the development of the optimal solution and the average cost solutions based on generations.

It is clear that the figure shows the reduction of **F** between the optimum value and the average value by generations, especially in the beginning and the frequent rebounding average value.

The initial sharp reduction proves that our algorithm works and as convergence, while the frequent rebound is due to the significant value of the mutation which causes a departure from the optimal solution for return.

This is the solution to avoid falling into a local minimum but by determining the speed of the algorithm.

According to and as for the figures (3.11), (3.12) and (3.13), we can say that the optimization results obtained by the genetic algorithm are calculated in accordance with the technical requirements imposed by the PF, in our grid where the voltages at all the buses are in the allowed range:

$$V_{i \min} \leq V_i \leq V_{i \max}$$

Same thing for the generated powers:

$$P_{g \min} \leq P_g \leq P_{g \max}$$

Transited powers are below their minimum value where the grid avoids the heating on the lines.

**Note:**

We must notice that the respect of the constraints imposed to the grid is ensured by the calculation of the PF for the studied cases in this thesis.

**2-2 PSO Method**

**2-2.1 Choice of parameters**

- Population size: 100
- Number of iterations: 20
- Number of lines: 41.

**2-2.2 Cost function**

We start first with the generation cost function. After the optimization program execution with the previous indicated parameters, we obtained these coming results:

**Table 3.5:** Optimized cost function and non-optimized losses function values.

<b>Optimized cost function value (Dollars/Hour)</b>	<b>Non-optimized losses function value (MW)</b>
801.8465	9.4125

And the generated power values corresponding to those values are:

**Table 3.6:** Optimal repartition of generated powers.

<b>Optimal repartition of generated powers for each production unit.</b>	
Generator Bus (N°)	Generated Power (MW)
1	177.2961
2	48.6500
5	21.4972
8	21.3584
11	12.0107
13	12.0000

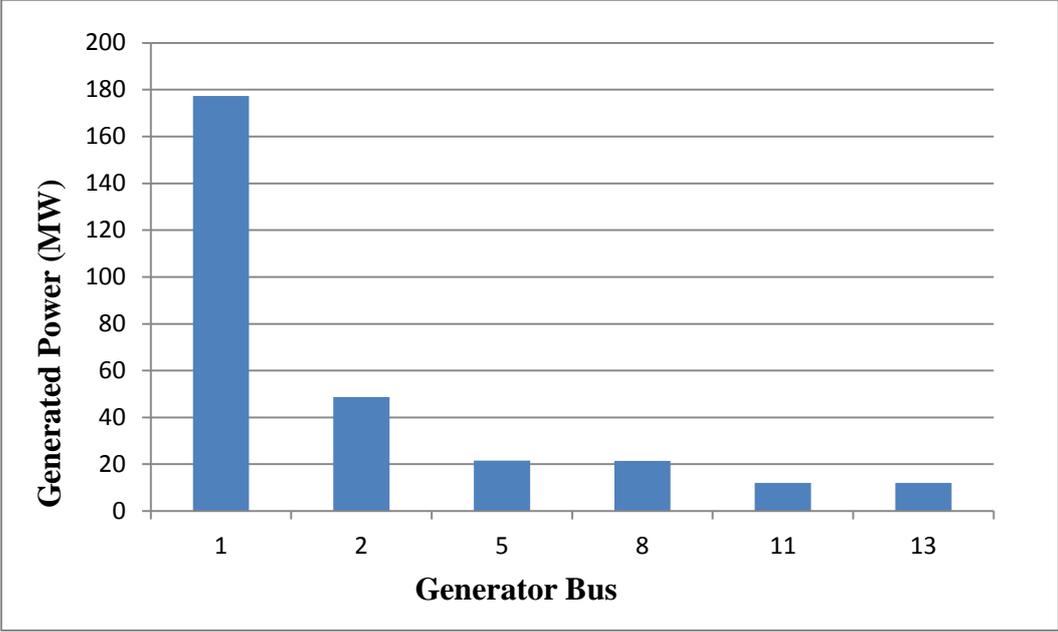


Figure 3.15: Optimal repartition of generated powers.

The calculation results are presented in the next figure:

Elapsed time is:  $t = 41.394639$  s

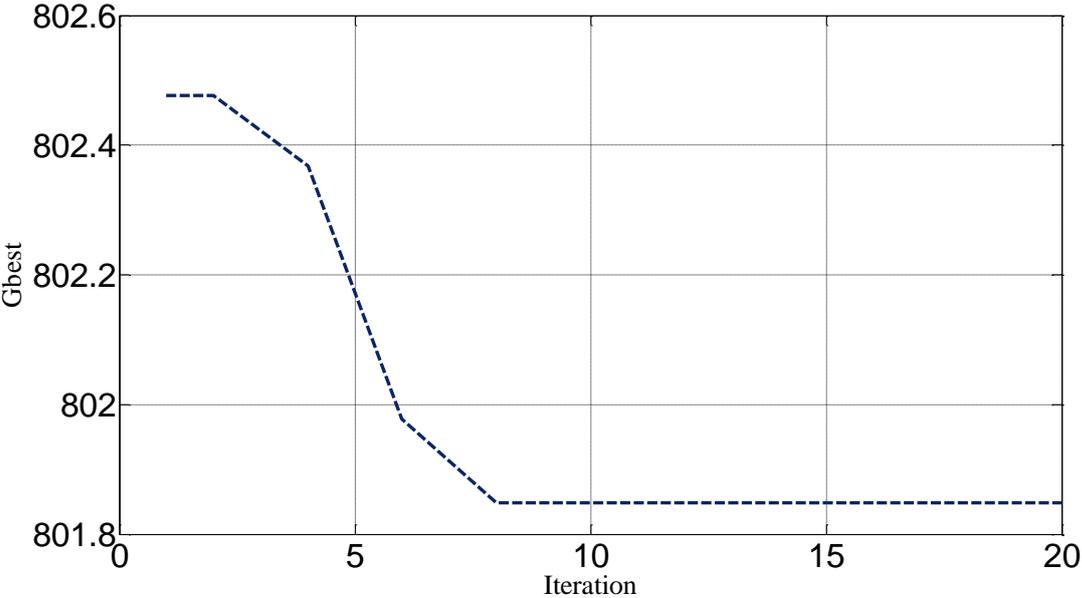


Figure 3.16: Evolution of the objective function over iterations.

2-2.2.1 Verification of constraints

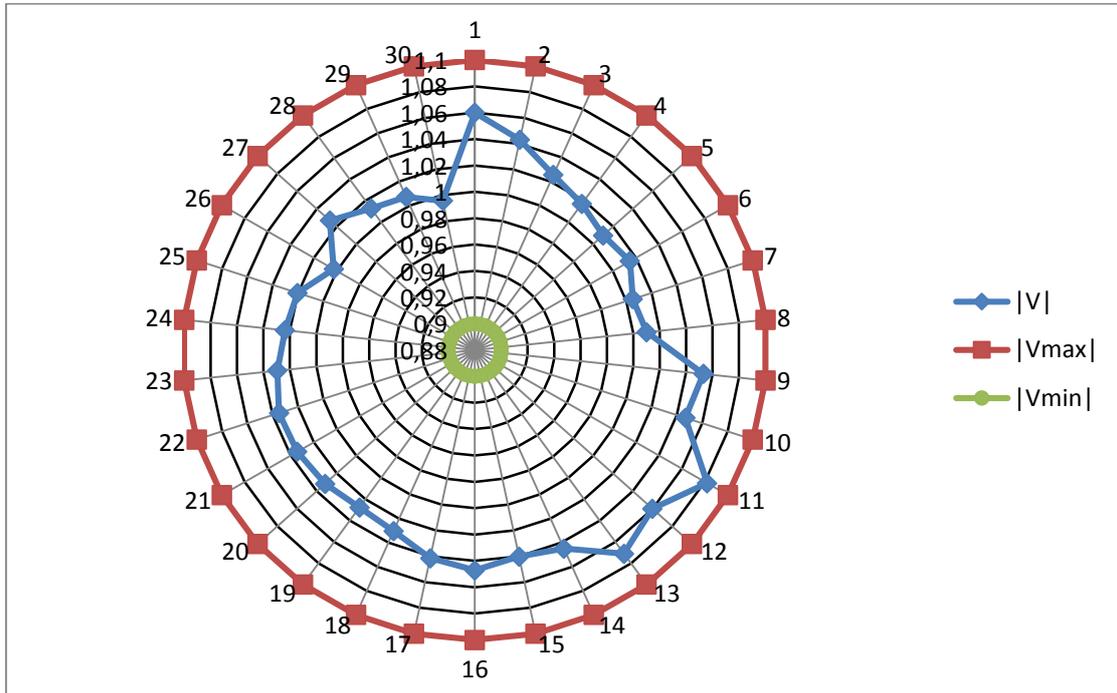


Figure 3.17: Verification of tensions constraint.

2-2.2.2 Discussion and Interpretation

The figure (3.15) shows the development of the optimal solution based on iterations.

According to and as for the figure (3.16), we can say that the optimization results obtained by the particle swarm optimization are calculated in accordance with the technical requirements imposed by the PF, in our grid where the voltages at all the buses are in the allowed range:

$$V_{i \min} \leq V_i \leq V_{i \max}$$

**Note:**

We must notice that the respect of the constraints imposed to the grid is ensured by the calculation of the PF for the studied cases in this thesis.

**2-2.3 Emission function**

Now, we move to the emission function. After the optimization program execution with the previous indicated parameters, we obtained these coming results:

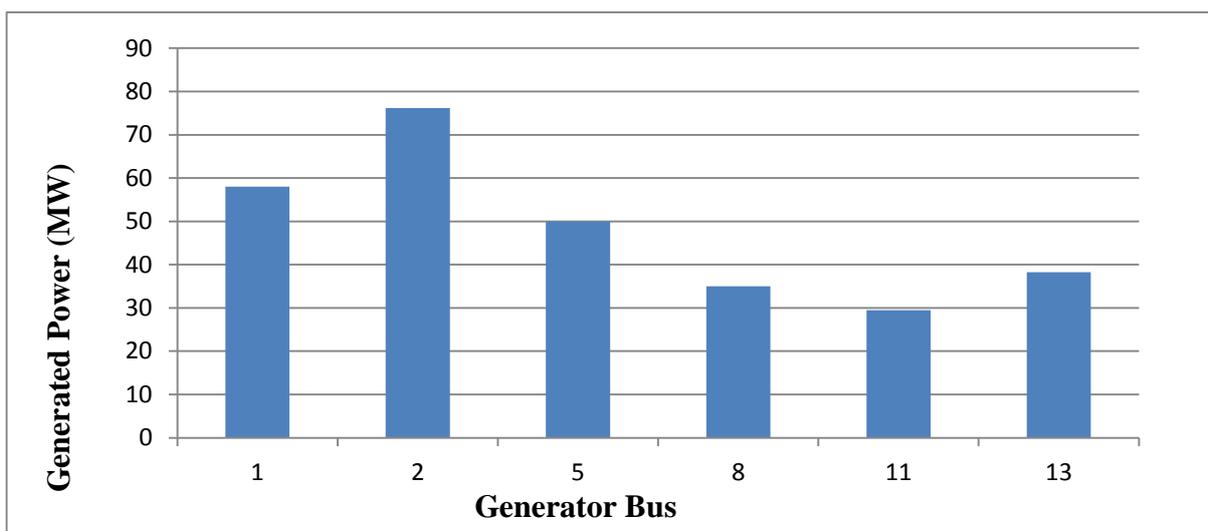
**Table 3.7:** Optimized emission function and non-optimized losses function values.

Optimized emission function value (Dollars/Hour)	Non-optimized losses function value (MW)
196.3030	3.5715

And the generated power values corresponding to those two values are:

**Table 3.8:** Optimal repartition of generated powers.

Optimal repartition of generated powers for each production unit.	
Generator Bus (N°)	Generated Power (MW)
1	57,9933
2	76,1979
5	50
8	35
11	29,5063
13	38,2739



**Figure 3.18:** Optimal repartition of generated powers.

The calculation results are presented in the next figure:

Elapsed time is:  $t = 42.726223$  s

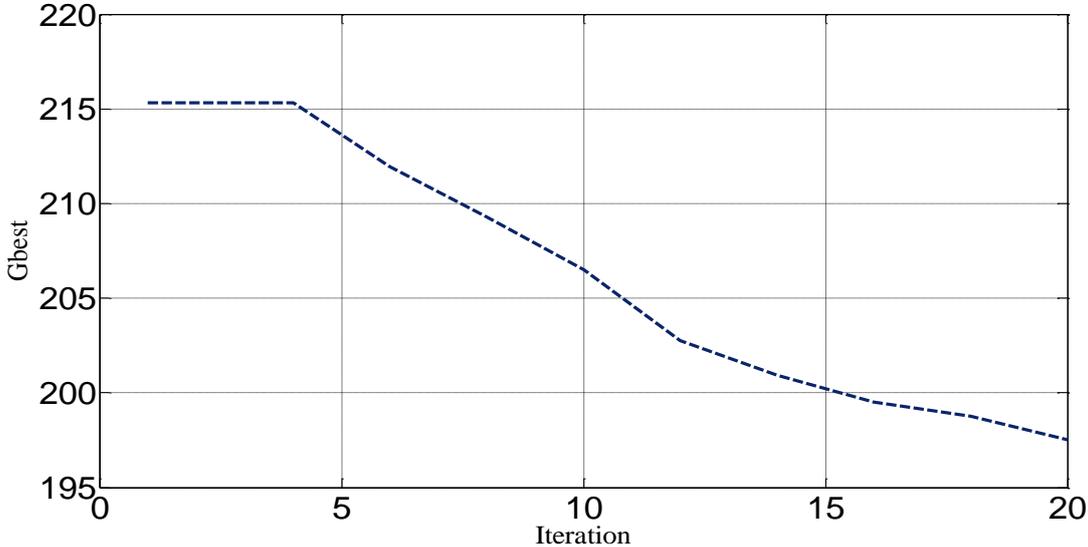


Figure 3.19: Evolution of the objective function over iterations.

2-2.3.1 Verification of constraints

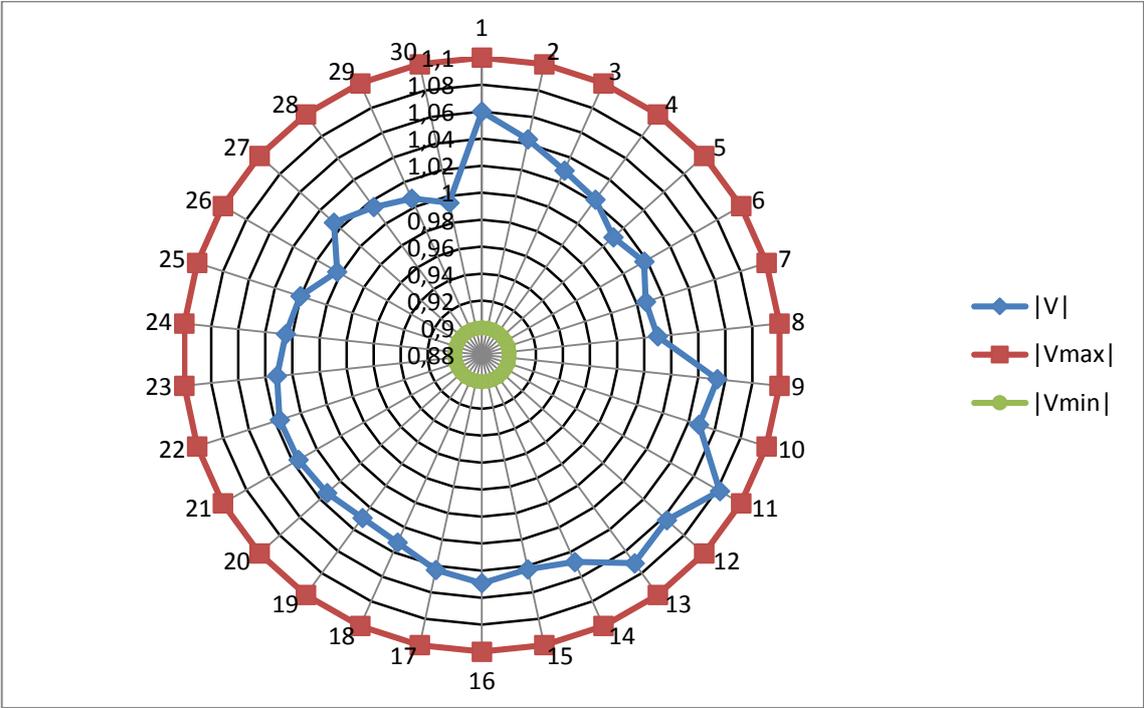


Figure 3.20: Verification of tensions constraint.

### **2-2.3.2 Discussion and Interpretation**

The figure (3.18) shows the development of the optimal solution based on iterations.

According to and as for the figure (3.19), we can say that the optimization results obtained by the particle swarm optimization are calculated in accordance with the technical requirements imposed by the PF, in our grid where the voltages at all the buses are in the allowed range:

$$V_{i \min} \leq V_i \leq V_{i \max}$$

#### **Note:**

We must notice that the respect of the constraints imposed to the grid is ensured by the calculation of the PF for the studied cases in this thesis.

#### **Conclusion**

In this final chapter, we tried to make a comparative analysis between two evolutionary methods: GAs and PSO, by applying them on a grid of 30 Bus to see their efficiency on the system. Before doing that, we tried to deal with two main functions which are cost function (economic dispatching) and also emission function (environmental dispatching).

# **General Conclusion & Perspectives**

## **General Conclusion and Perspectives**

The non-deterministic methods are there to solve the optimization problems where the conventional methods could not solve with the grid complexity, the non derivability or nonlinearity. Among these methods, we find: GA and PSO, which have similar characteristics and they are inspired from natural phenomena.

In this thesis, a comparative study was conducted to verify the performance of the two methods on the power grid by taking the same parameters.

The selected grid is the famous IEEE 30 Bus and the selected objective functions are: the cost function of production and the emission function of toxic gases, these two functions are represented by two non-linear expressions but with different degrees of non-linearity.

The different methods are programmed in MATLAB software taking into account the technical constraints imposed by the system, thanks to the PF algorithm (Newton-Raphson).

The program results showed a clear advantage for PSO in the computing-time side (41s versus 420s). But for the optimal values of the objective function, there is a slight improvement for the PSO against GA in a similar way for both objective functions.

We do not pretend that our work is exhausted. Therefore, we propose that the theme will serve as a basis for other comparisons with other different algorithms applied to a wide range of grid.

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# Appendix

## Appendix

- 30 Bus power grid.

## Bus data :

```
busdata=[1  1  1.06  0.0  0.0  0.0  0.0  0.0  0  0  0
          2  2  1.043 0.0 21.70 12.7 40.0 0.0 -40 50 0
          3  0  1.0  0.0  2.4  1.2  0.0  0.0  0  0  0
          4  0  1.06  0.0  7.6  1.6  0.0  0.0  0  0  0
          5  2  1.01  0.0  94.2 19.0  0.0  0.0 -40 40 0
          6  0  1.0  0.0  0.0  0.0  0.0  0.0  0  0  0
          7  0  1.0  0.0  22.8 10.9  0.0  0.0  0  0  0
          8  2  1.01  0.0  30.0 30.0  0.0  0.0 -10 60 0
          9  0  1.0  0.0  0.0  0.0  0.0  0.0  0  0  0
         10  0  1.0  0.0  5.8  2.0  0.0  0.0 -6 24 19
         11  2  1.082 0.0  0.0  0.0  0.0  0.0  0  0  0
         12  0  1.0  0  11.2  7.5  0  0  0  0  0
         13  2  1.071 0  0  0.0  0  0  -6 24 0
         14  0  1  0  6.2  1.6  0  0  0  0  0
         15  0  1  0  8.2  2.5  0  0  0  0  0
         16  0  1  0  3.5  1.8  0  0  0  0  0
         17  0  1  0  9.0  5.8  0  0  0  0  0
         18  0  1  0  3.2  0.9  0  0  0  0  0
         19  0  1  0  9.5  3.4  0  0  0  0  0
         20  0  1  0  2.2  0.7  0  0  0  0  0
         21  0  1  0  17.5 11.2  0  0  0  0  0
         22  0  1  0  0  0.0  0  0  0  0  0
         23  0  1  0  3.2  1.6  0  0  0  0  0
         24  0  1  0  8.7  6.7  0  0  0  0  4.3
         25  0  1  0  0  0.0  0  0  0  0  0
         26  0  1  0  3.5  2.3  0  0  0  0  0
         27  0  1  0  0  0.0  0  0  0  0  0
         28  0  1  0  0  0.0  0  0  0  0  0
         29  0  1  0  2.4  0.9  0  0  0  0  0
         30  0  1  0  10.6 1.9  0  0  0  0  0];
```

## Line data :

```
linedata=[1  2  0.0192  0.0575  0.02640  1
           1  3  0.0452  0.1852  0.02040  1
           2  4  0.0570  0.1737  0.01840  1
           3  4  0.0132  0.0379  0.00420  1
           2  5  0.0472  0.1983  0.02090  1
           2  6  0.0581  0.1763  0.01870  1
           4  6  0.0119  0.0414  0.00450  1
           5  7  0.0460  0.1160  0.01020  1
           6  7  0.0267  0.0820  0.00850  1
           6  8  0.0120  0.0420  0.00450  1
           6  9  0.0  0.2080  0.0  0.978
           6 10  0  .5560  0  0.969
           9 11  0  .2080  0  1
           9 10  0  .1100  0  1
           4 12  0  .2560  0  0.932
           12 13 0  .1400  0  1
           12 14 .1231 .2559 0  1
           12 15 .0662 .1304 0  1
           12 16 .0945 .1987 0  1
           14 15 .2210 .1997 0  1
           16 17 .0824 .1923 0  1
```

```

15 18 .1073 .2185 0 1
18 19 .0639 .1292 0 1
19 20 .0340 .0680 0 1
10 20 .0936 .2090 0 1
10 17 .0324 .0845 0 1
10 21 .0348 .0749 0 1
10 22 .0727 .1499 0 1
21 22 .0116 .0236 0 1
15 23 .1000 .2020 0 1
22 24 .1150 .1790 0 1
23 24 .1320 .2700 0 1
24 25 .1885 .3292 0 1
25 26 .2544 .3800 0 1
25 27 .1093 .2087 0 1
28 27 0 .3960 0 0.968
27 29 .2198 .4153 0 1
27 30 .3202 .6027 0 1
29 30 .2399 .4533 0 1
8 28 .0636 .2000 0.0214 1
6 28 .0169 .0599 0.065 1];

```

**Generation cost :**

```

gencost = [1 0.00375 2 0 50 200;
2 0.0175 1.75 0 20 80;
5 0.0625 1 0 15 50;
8 0.0083 3.25 0 10 35;
11 0.025 3 0 10 30;
13 0.025 3 0 12 40];

```

**Generation emission :**

```

%N° gama beta alfa epsil lambda Pmin Pmax
genemission=[1 6.49e-3 -5.554e-1 4.091e1 2e-4 2.857 50 200;
2 5.638e-3 -6.047e-1 2.543e1 5e-4 3.333 20 80;
5 4.586e-3 -5.094e-1 4.258e1 1e-6 8 15 50;
8 3.38e-3 -3.550e-1 5.426e1 2e-3 2 10 35;
11 4.586e-3 -5.094e-1 4.258e1 1e-6 8 10 30;
13 5.151e-3 -5.555e-1 6.131e1 1e-5 6.667 12 40];

```